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The Risk of Inflation Dispersion in the Euro Area

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Abstract

We introduce an approach to measure the risk of inflation dispersion among euro area countries. Our measure reflects the dissimilarity between the full predictive inflation distributions of member countries, and thus captures how "far" apart inflation levels are expected to be. The risk of inflation dispersion exhibits a countercyclical behavior along the business cycle. We document that the rising risk of inflation dispersion is mainly driven by a deterioration in financial conditions, while a robust anchoring of inflation expectations in each country tends to mitigate this risk. We further demonstrate that our measure has predictive power for future euro area inflation realizations as well as for variations in the monetary authority's interest rate.

Keywords: Inflation Dispersion, Kullback-Leibler, Euro area, Quantile Regression, Phillips Curve

JEL classification: D80, E31, E58, F45, G12

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NON-TECHNICAL SUMMARY

The resurgence of inflation in the euro area, spurred by the post-COVID crisis recovery and tensions in oil and natural gas markets following the war in Ukraine, has coincided with a significant increase in inflation differentials between member countries, reaching unprecedented levels and all-time highs.

Inflation divergence has drawn attention of policymakers since it can challenge the effectiveness of monetary policy. Adjusting nominal interest rates based on a single inflation target might result in an excessively accommodative monetary policy for nations experiencing notably higher inflation than the euro area average. Conversely, countries with inflation considerably below the average may encounter unwarranted tightening policy pressures.

Accurately assessing the risk of inflation dispersion is thus of deep relevance for policymakers. A key consideration is to determine whether inflation differentials are temporary or have the potential to persist over an extended period. Simultaneously, gaining insights into the drivers of inflation disparities provides valuable perspectives for predicting their future evolution and evaluating the risks associated with sustained divergences. Moreover, in the context of a monetary union, where countries share the same nominal interest rate, a high dispersion of inflation rates translates into a high dispersion of real interest rates between member countries. In this context, the risk of inflation dispersion may also be informative of the risk of financial fragmentation in the euro area, which may impair the transmission of monetary policy. However, the lack of reliable and available tools for analyzing these risks makes investigation on this important issue very limited. Indeed, current available measures of inflation dispersion typically rely on realized inflation, and thus, by construction, do not contain any forward-looking information about expected inflation dispersion at medium and long-term horizons.

In order to properly build a measure of expected inflation dispersion among euro area countries, one needs to take a probabilistic forecasting approach by considering not only cross-country differences in point forecasts of inflation, but also cross-country differences in density forecasts since they bring additional information, namely differentials in uncertainty and tail risks. This is what we intend to do in this paper. More specifically, our dispersion measure reflects the dissimilarity, i.e., the distance, between the full predictive inflation distributions of euro area member countries. Therefore, it captures how "far" apart inflation levels are expected to be between euro area members for a given horizon.

We provide evidence that the risk of inflation differentials shows a strong countercyclical pattern, which has tendency to rapidly increase during economic downturns (Figure 1). Our counterfactual analyses show that the rising risk of dispersion appears to be mainly associated with a deterioration in financial conditions. By contrast, a robust anchoring of inflation expectations tends to mitigate this risk. Finally, we demonstrate that our measure has predictive power for future euro area inflation realizations as well as for variations in the monetary authority's interest rate.

Figure 1. The Risk of Inflation Dispersion at 12 months horizon

Note: KL (Kullback-Leibler) divergence of one-year-ahead predictive inflation distributions of euro area countries. Gray shaded areas indicate CEPR-dated recessions.

Le risque de dispersion de l'inflation dans la zone euro

RÉSUMÉ

Nous présentons une approche pour mesurer le risque de dispersion de l'inflation entre les pays membres de la zone euro. Notre mesure reflète la dissemblance entre les distributions prédictives d'inflation des pays membres, et capte donc la « distance » attendue entre les niveaux d'inflation. Le risque de dispersion de l'inflation présente un comportement contra-cyclique tout au long du cycle économique. Nous montrons que l'augmentation du risque de dispersion de l'inflation est principalement due à une détérioration des conditions financières, alors qu'un ancrage solide des anticipations d'inflation dans chaque pays tend à atténuer ce risque. Nous démontrons en outre que notre mesure a un pouvoir prédictif pour les réalisations futures de l'inflation dans la zone euro, ainsi que pour les variations du taux d'intérêt de l'autorité monétaire.

Mots-clés : dispersion de l'inflation, Kullback-Leibler, zone euro, régression quantile, courbe de Phillips

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I. Introduction

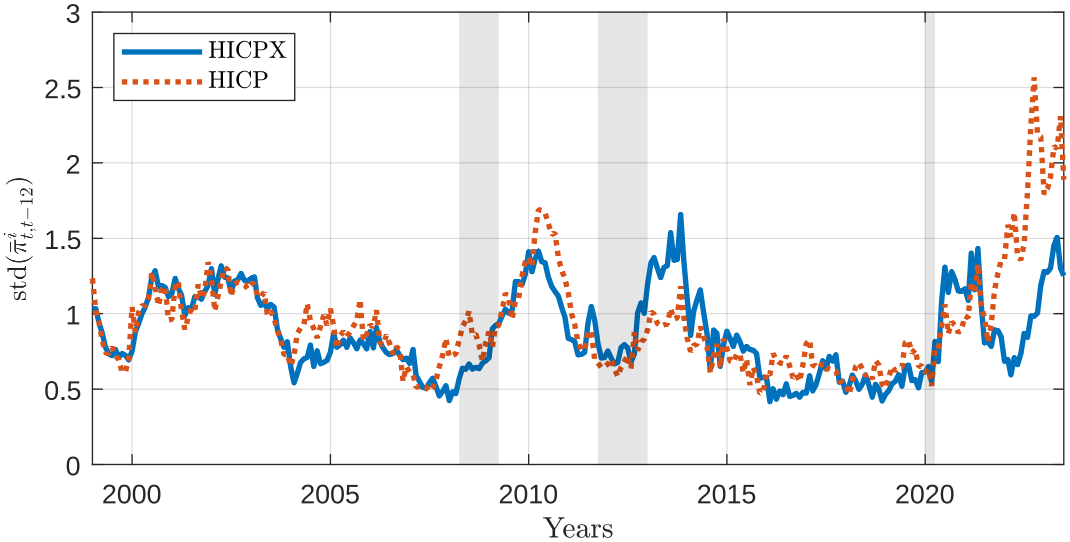
The resurgence of inflation in the euro area, spurred by the post-COVID crisis recovery and tensions in oil and natural gas markets following the war in Ukraine, has coincided with a significant increase in inflation differentials between member countries. These differentials, measured by the cross-country standard deviation in annual inflation rates across euro area members, have reached either unprecedented levels (HICP) or near all-time highs (HICP excluding food and energy), as depicted in Figure 1. Inflation divergence has drawn attention of policymakers since it can challenge the effectiveness of monetary policy.[1] Adjusting nominal interest rates based on a single inflation target might result in an excessively accommodative monetary policy for nations experiencing notably higher inflation than the euro area average. Conversely, countries with inflation considerably below the average may encounter unwarranted tightening policy pressures.

Accurately assessing the risk of inflation dispersion is thus of deep relevance for policymakers. A key consideration is to determine whether inflation differentials are temporary or have the potential to persist over an extended period. Simultaneously, gaining insights into the drivers of inflation disparities provides valuable perspectives for predicting their future evolution and evaluating the risks associated with sustained divergences. Moreover, in the context of a monetary union, where countries share the same nominal interest rate, a high dispersion of inflation rates translates into a high dispersion of real interest rates between member countries. In this context, the risk of inflation dispersion may also be informative of the risk of financial fragmentation in the euro area, which may impair the transmission of monetary policy. However, the lack of reliable and available tools for analyzing these risks makes investigation on this important issue very limited. Indeed, current available measures of inflation dispersion typically rely on realized inflation, as illustrated in Figure 1, and thus, by construction, do not contain any forward-looking information about expected inflation dispersion at medium and long-term horizons.[2]

[1]For example, in the monetary policy statement of July, 27$^{\text{th}}$ 2023, Christine Lagarde expressed her concern about the heterogeneity in inflation between euro are members: "*The numbers that we see now for Spain, with inflation trending towards 2% and hopefully sustainably so, plus unemployment numbers that are as low as they have ever been, is a good set of numbers for the country and for the economy at large. It is not the same for all Member States and there are Member States where inflation is still very high and has been high and is expected to remain high for longer. So we have to be very attentive to the aggregate numbers. Those are the ones that are driving our inflation outlook, helping us determine our policy. But we also have to look at each Member State and the characteristics of each Member State. We shall see.*"

[2]The ECB regularly publishes measures of inflation dispersion for the euro area using realized inflation. See, for example, Issing et al., 2003 and Consolo et al. (2021).

FIGURE 1. Cross-sectional Standard Deviation of Inflation in the Euro Area



Note: The figure shows the cross-country unweighted standard deviation of annual inflation rates in the euro area. $\bar{\pi}^i_{t,t-12}$ denotes the average over the last twelve months of the monthly inflation rate (core and headline inflation rates, annualized) for the country i of the euro area (Twelve countries, fixed composition, Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain). HICP stands for Harmonised Index of Consumer Prices, and HICPX stands for HICP inflation excluding energy and food. The sample is January 1999 to July 2023. Grey shaded areas indicate CEPR-dated recessions.

In order to properly build a measure of expected inflation dispersion among euro area countries, one needs to take a probabilistic forecasting approach by considering not only cross-country differences in point forecasts of inflation, but also cross-country differences in density forecasts since they bring additional information, namely differentials in uncertainty and tail risks. This is what we intend to do in this paper. More specifically, our dispersion measure reflects the dissimilarity, i.e., the distance, between the full predictive inflation distributions of euro area member countries. Therefore, it captures how "far" apart inflation levels are expected to be between euro area members for a given horizon. While our primary focus is on assessing expected inflation dispersion over a twelve-month horizon, we also consider shorter and longer horizons ranging from three-months to two-years ahead.

We employ a two-step method to estimate flexible parametric predictive distributions that account for skewness and heavy tails in inflation series.[3] The first step estimates the distributions semi-parametrically using quantile Phillips curve regressions for the first twelve euro area members, and in which inflation drivers are unemployment gap, oil price, financial stress,

[3]For empirical evidence of higher moments dynamics in inflation, see for example López-Salido and Loria (forthcoming).

past and expected inflation rates.[4] In the second step, each estimated quantile distribution is smoothed, each month, by interpolating between the estimated quantiles using the flexible skewed t-distribution along the lines of the work of Adrian, Boyarchenko, and Giannone (2019) on GDP growth, and more recently López-Salido and Loria (forthcoming) on inflation. This enables us to convert each empirical quantile distribution into an estimated conditional distribution of inflation. Then, we apply a generalization of the Kullback and Leibler (1951) divergence (denoted KL divergence, hereafter) for calculating the average divergence between all predictive distributions. The resulting series captures the expected inflation divergence between euro area members, and thus reflects the risk of inflation dispersion among euro area countries.

Based on our measure of expected divergence at the twelve-month horizon, we provide evidence that the risk of inflation differentials shows a strong countercyclical pattern, which has tendency to rapidly increase during economic downturns. Furthermore, the risk of dispersion demonstrates a consistent rise in the near and medium term, peaking at 18 months before gradually diminishing. Notably, during economic contractions, medium-term KL divergence shows greater sensitivity compared to its short-term counterpart.

We examine the sources contributing to this risk across three dimensions: quantile, inflation driver, and country. First, the risk of inflation dispersion arises more, on average, from variations in differences in the left tails of predictive inflation distributions than in the right tails. There are however specific periods like the Great Recession, where differences are due to other locations of the distributions. Second, the rising risk of dispersion appears to be mainly associated with a deterioration in financial conditions. By contrast, a robust anchoring of inflation expectations tends to mitigate this risk. Third, we show that no single country is the only source of dispersion risks on average over time, but some countries play a significant role in very specific episodes, like during the sovereign debt crisis and the COVID-19 crisis.

We also demonstrate that the measure of divergence of predictive distributions contains information about future inflation outcomes. After accounting for a range of macroeconomic and financial factors, we find that the risk of inflation dispersion improves inflation forecasts up to two-years ahead. According to our baseline specification, a one standard deviation increase in the risk of inflation dispersion predicts a 0.3 percentage points increase in inflation at horizons of twelve and twenty-four months. Depending on the horizon, the root mean square error (RMSE) of the forecast is reduced by 9% to 13% when contrasted with a model that omits our measures of the risk of divergence.

[4]Euro area countries used are: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain.

Finally, we find that the interest rates set by monetary authorities in the euro area partly respond to fluctuations in the risk associated with inflation differentials. Specifically, the effect is quantitatively significant in our regressions based on either shorter horizons up to one-year ahead: a one standard deviation increase in the measure of inflation dispersion is found to decrease the target interest rate by 6 to 13 basis points. The findings remain consistent when controlling for measures of inflation and output expectations. Thus, our estimation results suggest that the ECB responds to the risk of inflation dispersion beyond projections for the future level of aggregate inflation.

Relation to other studies. Our paper is related to the literature on inflation differentials, which has been a long-standing issue in the European Monetary Union.[5] Inflation dispersion was an important issue in defining the ECB's strategy at its inception (e.g., Issing et al., 2003), as well as in the recent ECB's strategy review in 2021 as discussed in depth by Consolo et al. (2021) and Reichlin et al. (2021).[6] From a theoretical perspective, Benigno (2004), Benigno and López-Salido (2006) and Kekre (2022) characterize the optimal monetary policy in a currency union with heterogeneity between countries. Galí and Monacelli (2008), Duarte and Wolman (2008), and Ferrero (2009) consider the role of optimal fiscal policy in the analysis. From an empirical perspective, the literature has been mainly focused on the underlying causes of realized inflation differentials in the euro area. Notable examples include Angeloni and Ehrmann (2007), Beck, Hubrich, and Marcellino (2009), and Estrada, Galí, and López-Salido (2013). More recently, Checherita-Westphal, Leiner-Killinger, and Schildmann (2023) empirically study the role of fiscal policy on inflation differentials in the EMU. We revisit this literature using our forward-looking measure of expected inflation divergence between euro area countries, which contain information not covered by divergence measures using realized inflation.

We also contribute to the literature on the estimation of the Phillips curve in the euro area. Galí, Gertler, and López-Salido (2001) show that standard Phillips curve fits euro area data very well. Ball and Mazumder (2021) reveal that a non-negligible role of inflation expectations and output gap in driving core inflation fluctuations in the euro area. Eser et al. (2020) give a broad picture of the implication of the Phillips curve analysis in the euro area for the conduct of ECB's monetary policy. In line with our paper, Baba et al. (2023) study the key drivers of the 2020-22 inflation surge across Europe and its dispersion across countries. All of these study examine the response of the conditional mean of euro area

[5] Haan (2010) offers a survey of this abundant literature subsequent to the creation of the euro area.

[6] As often reminded by the ECB, inflation differentials per see may not be detrimental to the monetary union if they reflect the process of nominal convergence and economic development catch up.

inflation to economic conditions. Our paper offers evidence that economic factors are still at work in the tails, but in a heterogeneous way between euro area countries.

Our paper falls also within the growing body of literature studying macroeconomic risks initiated by Adrian, Boyarchenko, and Giannone (2019); see also among others Plagborg-Møller et al. (2020), Figueres and Jarociński (2020), Adrian et al. (2022), Hilscher, Raviv, and Reis (2022), and López-Salido and Loria (forthcoming). Adrian, Boyarchenko, and Giannone (2019) estimate the conditional distribution of U.S. GDP growth as a function of economic and financial conditions using quantile regressions.[7] While this literature has focused on the predictive distributions of one single economic variable (such as GDP growth or inflation) with a particular emphasis on tail risks, we extend it in some way to the question of the heterogeneity of these distributions between countries by building our measure of KL divergence. Interestingly, Korobilis and Schröder (2023) develop a multicountry quantile factor augmented vector autoregression (QFAVAR) to capture heterogeneities both across euro area countries and across characteristics of the predictive distributions. However, the question of the degree of divergence is not addressed.

The rest of the paper is organized as follows. Section II presents quantile Phillips curve for each euro area countries, and discuss cross-country dispersion of parameters. Section III present and apply our approach to measure the risk of inflation dispersion. Section IV discusses the sources of inflation disperion risk by quantile, by inflation driver, and by country. Section V presents the policy implications. Section VI concludes.

II. Quantile National Phillips Curve

II.1. **Phillips Curve Quantile Regressions.** We rely on quantile regression models for studying the determinants of cross-country dispersion of the entire distribution of inflation. We follow the empirical strategy developed by López-Salido and Loria (forthcoming).[8] The key difference is that we apply this strategy to the first twelve countries of the euro area, instead of the euro area as a whole.

Let us denote by $\bar{\pi}^i_{t+1,t+h}$ the annualized average growth rate of Harmonized Index of Consumer Prices excluding food and energy (HICPX) between $t+1$ and $t+h$ for country i, and by x^i_t a $1 \times k$-dimensional vector containing the conditioning variables for country i, including a constant. Our benchmark horizon is $h = 12$, that is the average inflation over the next year. We consider a linear model for the conditional inflation quantiles whose predicted

[7] Macroeconomic tail risks can also be studied through the lens of Markov-switching models, as in Caldara et al. (2021) in the U.S. and Lhuissier (2022) in the euro area.

[8] The paper considers U.S economy, euro area but also a panel of OECD countries. See also Busetti, Caivano, and Rodano (2015) and Chortareas, Magonis, and Panagiotidis (2012) for the estimation of quantile Phillips curve for the euro area as a whole.

value:

$$\hat{Q}_\tau(\bar{\pi}^i_{t+1,t+h}|x^i_t) = x^i_t \hat{\beta}^i_\tau, \tag{1}$$

is a consistent linear estimator of the quantile function of $\bar{\pi}^i_{t+1,t+h}$ conditional on x^i_t; where $\tau \in (0, 100)$ is the quantile expressed in percentage, $\hat{\beta}^i_\tau$ is a $k \times 1$-dimensional vector of estimated quantile-specific parameters. More specifically, our quantile regression model for inflation is as follows:

$$\begin{aligned}\hat{Q}_\tau(\bar{\pi}^i_{t+1,t+h}|x^i_t) = \hat{\mu}^i_\tau + \left(1 - \hat{\lambda}^i_\tau\right) \pi^{*,i}_{t-1} + \hat{\lambda}^i_\tau \pi^{LTE,i}_t \\ + \hat{\theta}^i_\tau \left(u^i_t - u^{*,i}_t\right) + \hat{\gamma}^i_\tau \left(\pi^{o,*}_t - \pi^{*,i}_t\right) + \hat{\delta}^i_\tau f^i_t,\end{aligned} \tag{2}$$

where all variables are monthly time series covering January 1999 through July 2023.[9] Data sources are presented in the online Appendix. We impose some constraints, based on the literature, and use the inequality constrained quantile regression method developed by Koenker and Ng (2005) for the estimation.

The variables $\pi^{*,i}_{t-1}$ and $\pi^{LTE,i}_t$ represent average inflation over the previous twelve months and a measure of long-term inflation expectations, respectively. The relative importance of both variables is determined by the parameter λ^i_τ, with $0 \leq \lambda^i_\tau \leq 1$, as in Galí and Gertler (1999), Blanchard, Cerutti, and Summers (2015) and López-Salido and Loria (forthcoming) among others. We use six- to ten-years-ahead inflation expectations from Consensus Economics as long-term inflation expectation series.[10]

Our second factor is the unemployment gap measured as the difference between the unemployment rate u^i_t and the natural rate of unemployment $u^{*,i}_t$, which is obtained by applying the HP filter to the unemployment rate with the smoothing parameter equal to 14,400. The parameter θ^i_τ captures the slope of the Phillips curve at various inflation quantiles. Following Blanchard, Cerutti, and Summers (2015), we impose $\theta^i_\tau \leq 0$.

The third factor $\pi^{o,*}_t - \pi^{*,i}_t$ represents variations in relative oil price, where $\pi^{o,*}_t$ is the average inflation over the previous twelve months of crude oil price. This allows to capture the pass-through of oil prices into core inflation measures.[11] Our approach captures the effects of oil prices not only on the conditional mean of inflation, but on the entire inflation

[9]Our sample size aligns closely with other empirical studies examining the relationships between macroeconomic tail risks and financial conditions in the euro area. Notable examples include Figueres and Jarociński (2020) and López-Salido and Loria (forthcoming).

[10]As an alternative, inflation-linked swap (ILS) rates could be useful for deriving market-based measures of long-term inflation expectations. However, they are only available since 2004.

[11]Blanchard, Cerutti, and Summers (2015) consider import-price inflation in their estimated Phillips curve, that is proxied by oil price inflation at a monthly frequency in López-Salido and Loria (forthcoming). We also consider commodity and energy prices instead of oil price using the above-described specification of the augmented quantile Phillips curve. The results are robust to the choice of the series and are not reported here.

distribution. Cross-quantile and cross-country variations in the parameters γ_τ^i in equation (2) capture its effects. Here again, we follow Blanchard, Cerutti, and Summers (2015) and impose $\gamma_\tau^i \geq 0$.[12]

The fourth factor f_t^i represents financial conditions. The literature has documented firms financing conditions also helps to explain inflation dynamics. Notable examples include Del Negro, Giannoni, and Schorfheide (2015), Christiano, Eichenbaum, and Trabandt (2015) and Gilchrist et al. (2017). More importantly, López-Salido and Loria (forthcoming) extend the analysis to consider the effect of financial conditions on the conditional distribution of inflation. Following these authors, we approximate f_t^i by the Composite Indicator of Systemic Stress (CISS), except for Luxembourg for which we use the Country-Level Index of Financial Stress (CLIFS).[13] The parameter associated with financial conditions in our empirical specification of the Phillips curve is δ_τ^i. This coefficient is left unconstrained in that case, since no consensus has been reached in the literature regarding the effect of financial conditions on the overall inflation distribution.

II.2. **Cross-country Heterogeneity of Phillips Curve parameters.** Figure 2 synthesizes the estimated coefficients across quantiles and countries.[14] For each variable, it shows two different kinds of information on estimated coefficients. The first one is about the magnitude of the coefficient and the second one is about its cross-country dispersion.

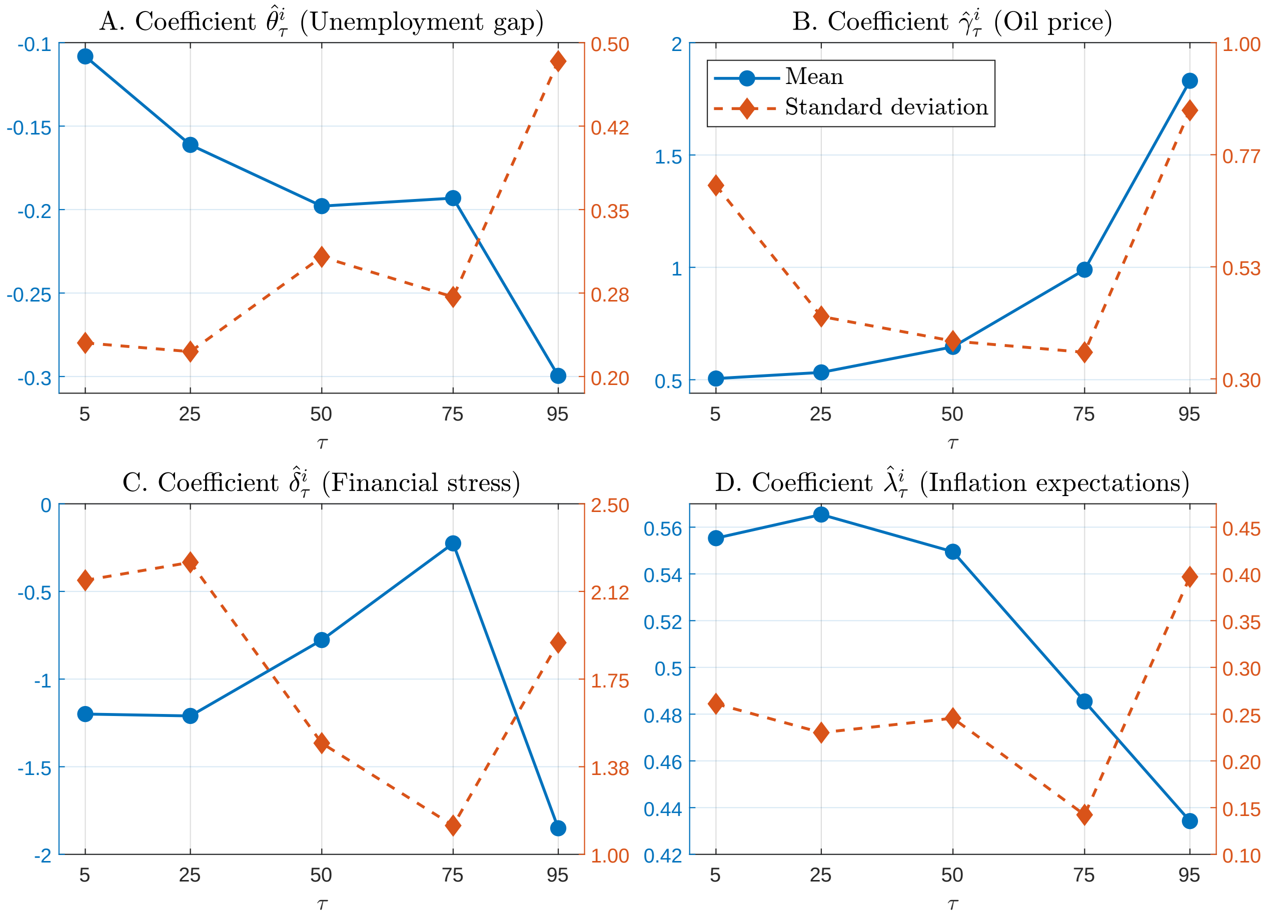
For the magnitude, we report the mean of the estimated coefficients for each quantile to determine to what extent the variable has a greater or lesser impact on inflation (in average for all countries) depending on the quantile considered. For the cross-country dispersion, we report the standard deviation of the estimated coefficients for each quantile to determine to what extent the impact of this variable on inflation is more or less dispersed across countries, depending on the quantile under consideration. Ultimately, we can identify the quantiles for which certain variables play an important role in inflation dynamics and are a source of structural heterogeneity between countries.

[12]The literature provides evidence of the role of energy price and import prices as a key inflation determinant. For instance, Kilian and Zhou (2021) find that gasoline prices do not explain the improved fit of the Phillips curve augmented by household inflation expectations during the years that followed the Great Recession. On the other hand, Matheson and Stavrev (2013) find an increasing importance of import-price in explaining inflation fluctuations, while Salisu, Ademuyiwa, and Isah (2018) point a better forecast performance when including oil prices into the Phillips curve.

[13]The CISS, developed by Kremer, Lo Duca, and Holló (2012), is a weekly index maintained by the ECB. It includes 15 raw series, mainly market-based financial stress measures that are split equally into five categories: financial intermediaries, money markets, equity markets, bond markets and foreign exchange markets. The CLIFS, proposed by Peltonen, Klaus, and Duprey (2015), follows the approach of the CISS, but with slightly different market segments.

[14]We provide a more complete description of results in the online Appendix.

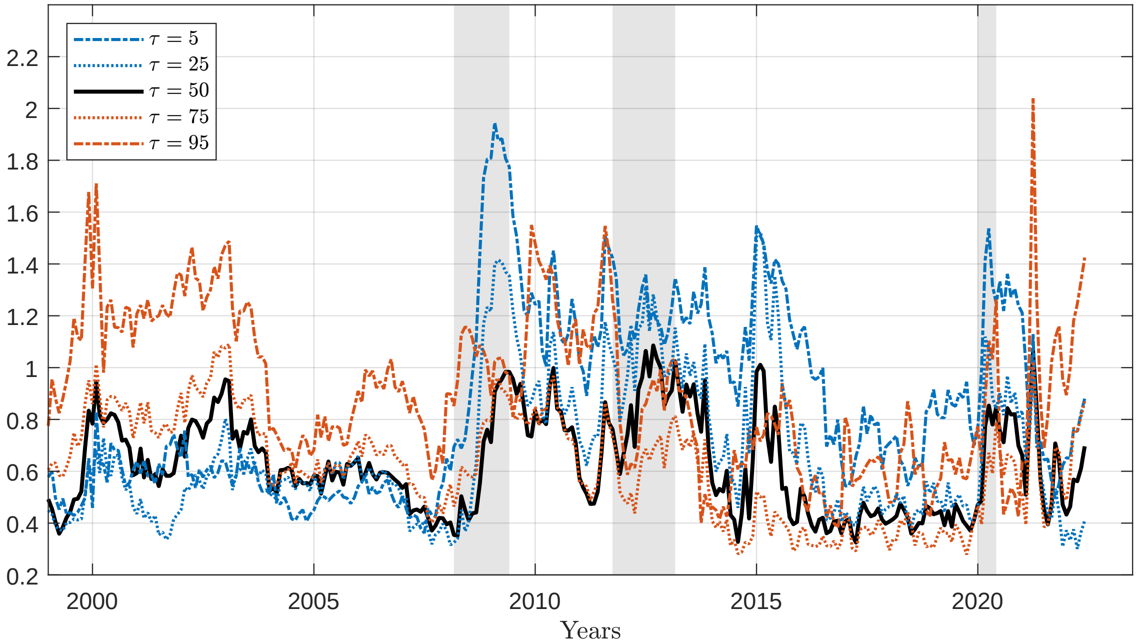
FIGURE 2. Estimated Coefficients by Quantile: Magnitude and Dispersion



Note: For each estimated coefficient of the quantile regression defined by equation (2), the figure reports the unweighted mean of the estimated coefficients (blue solid line with circles) and the standard deviation (red dashed line with diamonds). For each panel, the title panel gives the symbol of the coefficient and the associated variable in parenthesis.

For unemployment gap, the figure suggests a steeper Phillips curve for higher quantiles: the average value of coefficients reaches its maximum for $\tau = 10$, with a value of -0.09, and its minimum at the quantile $\tau = 95$, with a value of -0.29. The cross-country dispersion of the estimated coefficients is the highest for the top quantile ($\tau = 95$). Unemployment gap can therefore be considered as a potential source of inflation dispersion risk at the upper tail of the distribution. For energy prices, the magnitude of estimated coefficients increases with the quantile considered, with a sharp increase between $\tau = 50$ and $\tau = 95$ (from 0.64 to 1.83). Estimated coefficients are less dispersed in the middle of the distribution than in the tails. This suggests that energy prices can therefore be considered as a source of inflation dispersion for both downward and upward inflation risks. Regarding financial stress, the magnitude of estimated coefficients is the lowest for $\tau = 75$ and increases when we consider

Figure 3. Dispersion of Conditional Quantiles



Note: Standard deviation of conditional inflation quantiles $\hat{Q}_\tau(\bar{\pi}^i_{t+1,t+h}|x^i_t)$ across country i, for quantiles $\tau = \{5; 25; 50; 75; 95\}$ and forecast horizon $h = 12$. Grey shaded areas indicate CEPR-dated recessions.

extreme quantiles. The cross-country dispersion of estimated coefficients exhibits a U-shaped with high values extreme quantiles. This suggests that financial stress may be responsible of inflation dispersion especially both at the left and the right of predictive distributions. Finally, for inflation expectations, the magnitude of estimated coefficients decreases with the quantile while its cross-section dispersion increases. Since the complement of this coefficient, $(1 - \lambda)$, accounts for inflation inertia it means that past inflation can be a potential source of dispersion for upward risks associated to the quantile $\tau = 90$.

II.3. **Cross-country Heterogeneity of Quantile Regressions.** Figure 3 depicts the evolution of the dispersion of conditional quantiles across countries for the one-year forecast horizon. Two stylized facts emerge. First, there is strong evidence of time variation in the cross-country dispersion of conditional quantiles. For any quantile, the dispersion tends to increase during economic downturns like the Great Recession, the sovereign debt crisis, and the Covid-19 crisis. Clearly, the dispersion is countercyclical. Second, there are significant differences in magnitude of variations across quantiles over time. The cross-country standard deviation of the 50$^{\text{th}}$ quantile appears to be always smaller than either the lower or the upper quantiles throughout the sample. This means that particular attention must be paid on tail risks when investigating cross-country divergence. In particular, inflation dispersion is clearly higher for the upper quantiles (75$^{\text{th}}$ and 95$^{\text{th}}$) than for other quantiles during the

2001-2003 economic downturn, marked by the 9/11 terrorist attacks, Dot-com bubble, and corporate scandals. By contrast, from the Great Recession of 2008-09 to the COVID crisis, the situation is reversed. The highest inflation dispersions are associated with downside risk of inflation (5^{th} and 25^{th} quantiles).

To sum up, the evolution of the cross-country dispersion varies differently depending on the quantile. So focusing exclusively only on one particular quantile is not sufficiently informative about the degree of expected inflation dispersion among euro area countries. In the next section, we propose a unified measure that consider all quantiles of the distribution.

III. Measuring Divergence

This section relies on the quantile regression Phillips curve estimates to construct our measure of the risk of inflation dispersion. Section III.1 shows how we map the quantile regression estimates into a flexible distribution to recover a probability density function for each country. Section III.2 shows how our measure of divergence is computed using those distributions.

III.1. **The Conditional Inflation Distribution.**

The quantile regression (2) furnishes us with rough estimates of the quantile function, which represents an inverse cumulative distribution function. However, translating these estimates into a probability distribution function becomes challenging due to approximation errors and estimation noise. Following Adrian, Boyarchenko, and Giannone (2019), we map the quantile regression estimates into a skewed t-distribution to recover and show a probability density function. The skewed t-distribution was developed by Azzalini and Capitanio (2003) and has the following form:

$$f(\bar{\pi}^i_{t+1,t+h}|x^i_t,\mu^i_t,\sigma^i_t,\eta^i_t,\kappa^i_t) = \frac{2}{\sigma^i_t} t(z^i_{t,t+h};\kappa^i_t) T\left(\eta^i_t z^i_{t,t+h}\sqrt{\frac{\kappa^i_t+1}{\kappa^i_t+(z^i_{t,t+h})^2}};\kappa^i_t+1\right),$$

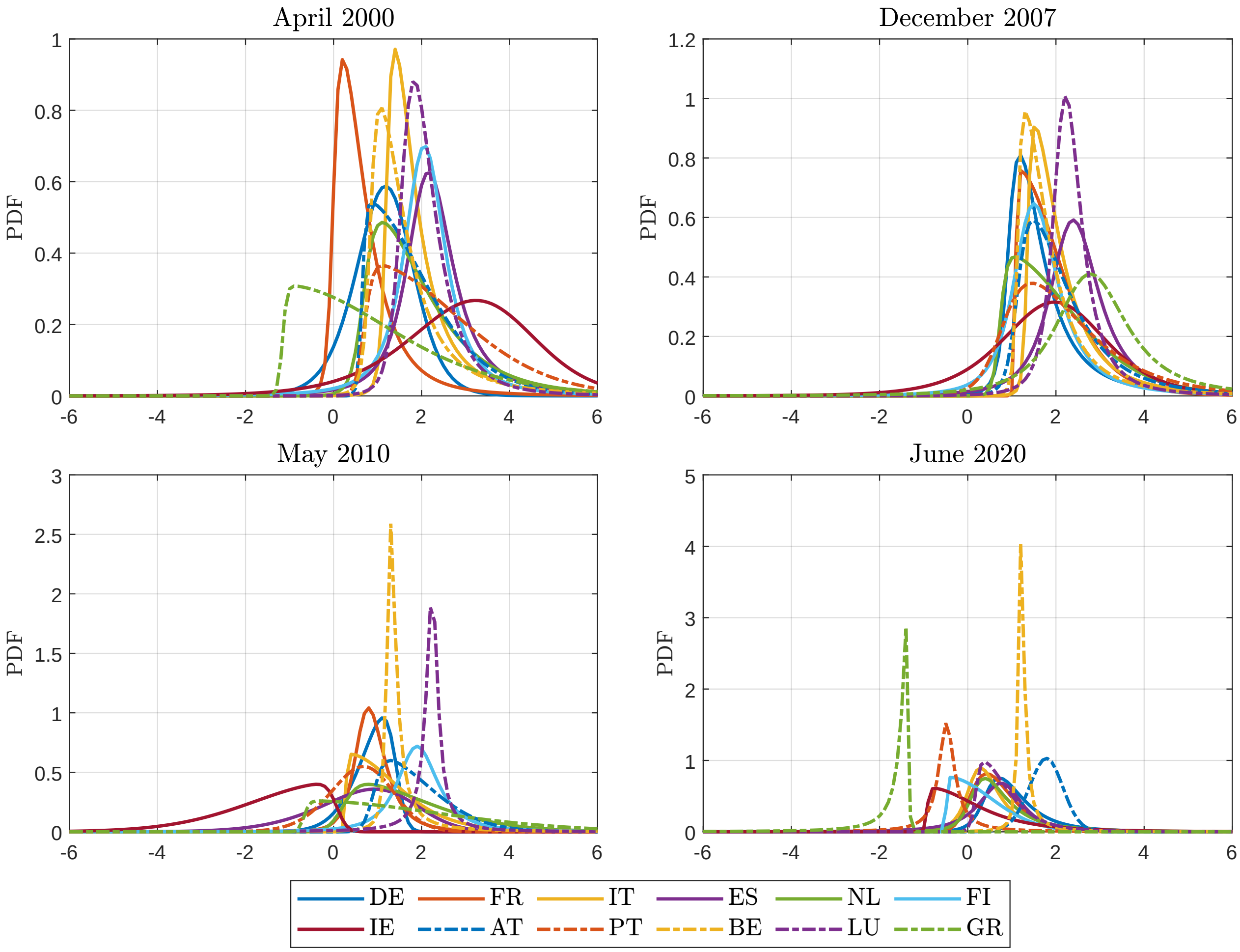
where $z^i_{t,t+h} = \frac{\bar{\pi}^i_{t+1,t+h}(x_t)-\mu^i_t}{\sigma^i_t}$, and t and T represent the density and cumulative distribution function of the student t-distribution, respectively. The four time-varying parameters of the distribution pin down the location μ^i_t, scale σ^i_t, shape η^i_t, and fatness κ^i_t for each country i, where η^i_t and κ^i_t parameters control the skewness and the kurtosis of the distribution, respectively.

For each month and each country, we choose the four parameters (μ^i_t, σ^i_t, η^i_t, κ^i_t) of the skewed t-distribution to minimize the squared distance between our estimated quantile function $\hat{Q}_\tau(\bar{\pi}^i_{t+1,t+h}|x^i_t)$ obtained from the quantile Phillips curve model in equation (2) and the quantile function of the skew t-distribution to match the 5^{th}, 25^{th}, 75^{th} and 95^{th} quantiles.

As an illustration, Figure 4 plots the fitted conditional probability density functions of country-specific inflation for four sample dates at different points of the business cycle: April

2000, during the dotcom bubble burst; December 2007, which represented the end of the ECB's tightening cycle before the financial crisis; May 2010, when the Greece received its first bailout; and June 2020, which is the COVID-19 period.

FIGURE 4. One-year Ahead Predictive Densities



Note: The panels in this figure show the estimated skewed t-density functions for one-year-ahead country-specific inflation for four sample dates at different points of the business cycle: April 2000, December 2007, May 2010, and June 2020.

When comparing the conditional densities among different countries on a specific date, notable variations are observed in these densities. These differences stem not only from changes in the estimated values at the point forecast (at the mode) but also from variations in the upper and lower ends of the distributions. During the period of financial distress, marked by the dotcom bubble in the 2000s, some distributions take on a Gaussian shape and are mostly confined to positive values, while others exhibit smaller expected values, higher variance, and positive skewness.

Similarly, when comparing the conditional densities across four different dates, significant time variations in the entire distributions are observed. These variations, once again, arise from both changes in the point forecast and risks associated with the tails.[15] In December 2007, during the expansion of the business cycle, the predictive inflation distributions are concentrated around the ECB's two percent inflation target. In contrast, during economic slowdowns, like in April 2000, May 2010, or June 2020, the distributions appear considerably more dispersed, with greater variations in the point forecast, larger variance, compared to the more stable distribution observed in December 2007.

To accurately assess the risk of inflation dispersion, it is therefore crucial to consider cross-country differences in the complete distributions of future inflation, rather than focusing solely on the forecast midpoint. In the following section, we introduce a measure that accounts for this aspect.

III.2. **KL divergence.** This section aims to quantify the expected disparity in future inflation among euro area countries by measuring the dissimilarity between all predictive inflation distributions. This comprehensive approach allows us to provide a thorough assessment of the expected divergence between countries, considering not just point forecasts but the full predictive distributions.

More formally, we denote by $\hat{f}_{\hat{\pi}_{t+1,t+h}}(\bar{\pi}^i; x_t^i) = f(\bar{\pi}^i; \hat{\mu}_{t+h}^i, \hat{\sigma}_{t+h}^i, \hat{\alpha}_{t+h}^i, \hat{\nu}_{t+h}^i)$ the estimated conditional skew-t density in country i. We define the average divergence, $D_{KL,t}(h)$, at horizon h as

$$D_{KL,t}(h) = \frac{1}{N(N-1)} \sum_{i}^{N} \sum_{j}^{N} KL_{i,j,t}(h), \qquad \text{for} \quad i \neq j, \tag{3}$$

where

$$KL_{i,j,t}(h) = \int_{-\infty}^{\infty} \log\left(\frac{\hat{f}_{t+h}(\bar{\pi}^i; x_t^i)}{\hat{f}_{t+h}(\bar{\pi}^j; x_t^j)}\right) \hat{f}_{t+h}(\bar{\pi}^i; x_t^i) d\bar{\pi}^i, \tag{4}$$

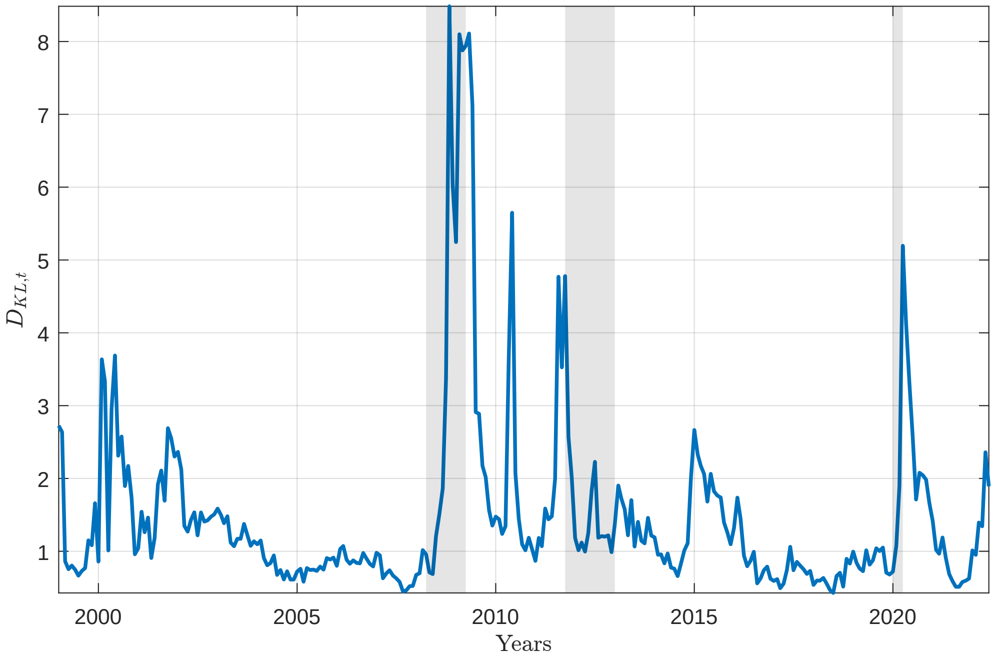
is the KL divergence,[16] which measures the divergence of $\hat{f}\,(\bar{\pi}^i)$ from $\hat{f}\,(\bar{\pi}^j)$ and where the expectation defined with respect to the density $\hat{f}(\bar{\pi}^i)$. This measure is always positive and is equal to zero if and only if $f(\bar{\pi}^i) = f(\bar{\pi}^j)$. Intuitively, KL measures the divergence between the conditional density of country i and the conditional density of country j. KL is considered as a good indicator of the correlation degree between two densities. For $N = 2$, the average divergence is fundamentally the divergence in the sense of KL. Our generalized KL divergence to multiple dimensions ($N \geq 2$) follows Sgarro (1981) and takes the average divergence of

[15]Figure C1 in the online Appendix shows changes over time in the dispersion across countries of the four moments of the skewed t-distribution.

[16]See Kullback and Leibler (1951).

distributions. Our measure can be interpreted as a sort of "directed distance" between all distributions.

FIGURE 5. The Risk of Inflation Dispersion $D_{KL,t}(h = 12)$



Note: The KL measure $D_{KL,t}(h = 12)$ of one-year-ahead predictive inflation distributions of euro area countries defined by equation (3). Gray shaded areas indicate CEPR-dated recessions.

A KL value of zero suggests no risk of inflation dispersion in the euro area, indicating that predicted inflations for each member are identical and drawn from the same predictive distributions. An increase (decrease) in the KL value reflects a divergence (convergence) in the predicted inflation distributions among euro area members, signifying a higher (lower) risk of inflation dispersion. In simpler terms, a greater (lower) KL value implies a higher (lower) likelihood that future realized inflation will vary significantly, based on the current dissimilarity (similarity) in predicted inflation distributions.

Figure 5 depicts our estimated measure of expected divergence in inflation among members of the euro area at horizon $h = 12$, denoted $D_{KL,t}(h = 12)$. Our indicator reveals a clear countercyclical pattern, tending to notably escalate during economic downturns. Examples of such downturns include the period from 2000 to 2002, characterized by events like the 9/11 terrorist attacks, the Dot-com bubble, and corporate scandals, the Great Recession in 2008-09, the sovereign debt crisis in 2010-12, and the COVID-19 recession. Notably, the peaks in divergence appear most pronounced during the Great Recession, with KL divergence values nearly doubling those observed during the sovereign debt crisis or the COVID-19 recession. This suggests that financial conditions might serve as the primary driver behind the generation of inflation dispersion risk. We will confirm this intuition in the next section.

Table I. Mean and standard deviation of $D_{KL,t}(h)$ by horizon h

	Horizon h							
	$h = 3$	$h = 6$	$h = 9$	$h = 12$	$h = 15$	$h = 18$	$h = 21$	$h = 24$
Mean	0.88	1.18	1.27	1.61	1.69	1.72	1.65	1.46
Std. Dev.	0.66	0.88	1.06	1.39	1.32	1.24	1.05	0.99

Note: The table shows the mean and standard deviation of $D_{KL,t}(h)$ defined by equation (3) at horizons $h = 3, 6, \ldots, 24$ months over the sample period.

We generate the term structure of KL divergence to illustrate the evolution of the risk of inflation divergence across various time horizons. The process involves using quantile regression methods on our dataset consisting of twelve euro area countries to forecast periods ranging from three to twenty-four months. Subsequently, for each horizon and country, we empirically map the quantile regression estimates to the skewed t-distribution. Finally, we compute the average KL divergence for each horizon, representing the term structure of inflation divergence risk.

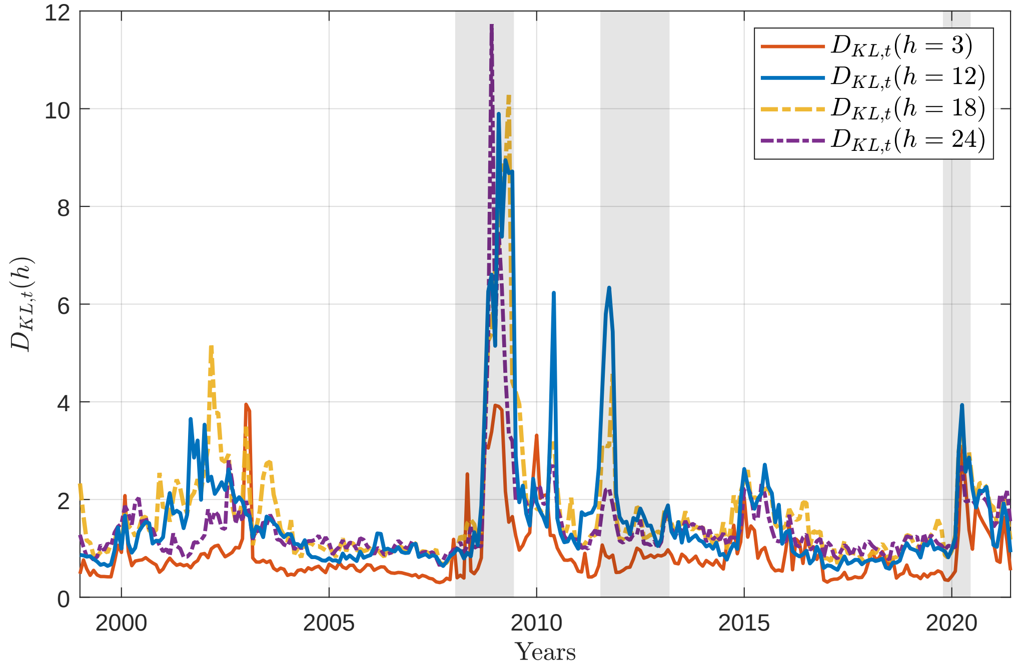
Table I displays the mean and the standard deviation of the resulting term structure for projection horizons up to two years. The risk of divergence seems to exhibit a steady increase in the near and medium term: its average over the sample period reaches its peak at eighteen-month horizon before declining. Regarding the standard deviation, KL divergence is increasingly volatile up to a one-year horizon before decreasing thereafter. Medium- and long-term KL divergences (for $h = 18$ or $h = 24$, respectively) appear to be more responsive during economic contractions compared to short-term KL divergence (for $h = 3$), as illustrated in Figure 6.

IV. Anatomy of Risk

In this section, we examine the sources contributing to the risk of increasing inflation divergence along three dimensions: quantile, economic factor, and country.

IV.1. **KL divergence across quantiles.** The KL measure used in this paper takes advantages of the entire predictive inflation distributions to measure the expected inflation divergence between euro area countries. In this section, we investigate whether the divergence is due to differences in the probability masses that the conditional distributions assign to specific range of quantiles of the distributions.

FIGURE 6. The Risk of Inflation Dispersion by Forecast Horizon



Note: The figure shows the evolution of KL divergence $D_{KL,t}(h)$ at horizons $h = 3$, 12, 18, and 24 defined by equation (3), over the sample period. Gray shaded areas indicate CEPR-dated recessions.

We define the average divergence between quantiles τ and $\tau + 10$, $D_{KL,t}^{[\tau,\tau+10]}(h)$, as

$$D_{KL,t}^{[\tau,\tau+10]}(h) = \frac{1}{N(N-1)} \sum_{i}^{N} \sum_{j}^{N} KL_{i,j}^{[\tau,\tau+10]}(h), \qquad \text{for} \quad i \neq j, \tag{5}$$

where

$$KL_{i,j,t}^{[\tau,\tau+10]}(h) = \int_{\hat{F}_{t+h}^{-1}(\bar{\pi}_t^{i,\tau})}^{\hat{F}_{t+h}^{-1}(\bar{\pi}_t^{i,\tau+10})} \log\left(\frac{\hat{f}_{t+h}(\bar{\pi}^i; x_t^i)}{\hat{f}_{t+h}(\bar{\pi}^j; x_t^j)}\right) \hat{f}_{t+h}(\bar{\pi}^i; x_t^i) d\bar{\pi}^i, \tag{6}$$

with $\hat{F}_{t+h}^{-1}(.)$ is the cumulative distribution associated with $\hat{f}_{t+h}(.)$ and $\hat{F}_{t+h}^{-1}(\bar{\pi}_t^{i,\tau})$ is the level of inflation in country i associated to the τ-th quantile. To be consistent with our baseline measure, we set $h = 12$.

Table II reports the mean and the standard deviation of KL by quantiles $[\tau, \tau + 10]$. As can be seen, the risk of inflation dispersion stems more from variations in the left tails of predictive inflation distributions than from variations in the right tails. Notably, the mean of the 10^{th} quantile is almost twice as large as that of the 90^{th} quantile. Interestingly, the standard deviation is higher for KL measures at the tails of the distribution, namely $D_{KL,t}^{[0,10]}$ and $D_{KL,t}^{[90,100]}$. Although these statistics may mask disparities over time, they are nevertheless useful for providing information on the role of tails in the evolution of our baseline KL measure.

TABLE II. Mean and standard deviation of $D^{[\tau,\tau+10]}_{KL,t}$ by quantiles $[\tau, \tau + 10]$

	Quantiles $[\tau, \tau+10]$									
	$[0, 10]$	$[10, 20]$	$[20, 30]$	$[30, 40]$	$[40, 50]$	$[50, 60]$	$[60, 70]$	$[70, 80]$	$[80, 90]$	$[90, 100]$
Mean	0.22	0.19	0.18	0.17	0.15	0.14	0.12	0.10	0.09	0.12
Std. Dev.	0.14	0.11	0.11	0.11	0.12	0.13	0.14	0.16	0.18	0.24

Note: The table shows the mean and standard deviation of $D^{[\tau,\tau+10]}_{KL,t}(h = 12)$ defined by equation (5) for quantiles $\tau = 0, 10, \ldots, 90$ over the sample period.

The gap between KL measures by quantile is illustrated by Figure 7, which depicts the time series of quantile-based measures. Two main observations emerge. First, while the 10^{th} quantile is, on average, higher than other quantiles, specific periods exist during which other quantiles exhibit a higher risk of dispersion. For instance, during the Great Recession, $D^{[90,100]}_{KL,t}$ rose dramatically, whereas $D^{[0,10]}_{KL,t}$ remained at moderated levels. Second, our baseline measure contains information not captured by a simple divergence measure that focuses solely around point forecasts, such as $D^{[45,55]}_{KL,t}$, overlooking cross-country differences in uncertainty and tail risks. For example, during the early 2000s, $D^{[0,10]}_{KL,t}$ and $D^{[90,100]}_{KL,t}$ rapidly rose while $D^{[45,55]}_{KL,t}$ remained relatively stable. Therefore, neglecting cross-country differences in uncertainty and tail risks may distort the inference of the risk of inflation divergence between euro area countries.

IV.2. **KL divergence across drivers.** To gain an appreciation of the economic origins of the risk of inflation divergence described in the previous section, we investigate the role of inflation drivers contained in our Phillips curves. To do so, we proceed as follows. Let us consider one of the variables $j = 1, ..., J$ introduced as a driver of inflation in the quantile regression defined by equation (1). Since the regression is linear, we can rewrite equation (1) as follows:

$$\hat{Q}_\tau(\bar{\pi}^i_{t+1,t+h}|x^i_t) = x^i_{t,j}\hat{\beta}^i_{\tau,j} + \hat{Q}_{\tau,-j}(\bar{\pi}^i_{t+1,t+h}|x^i_t), \tag{7}$$

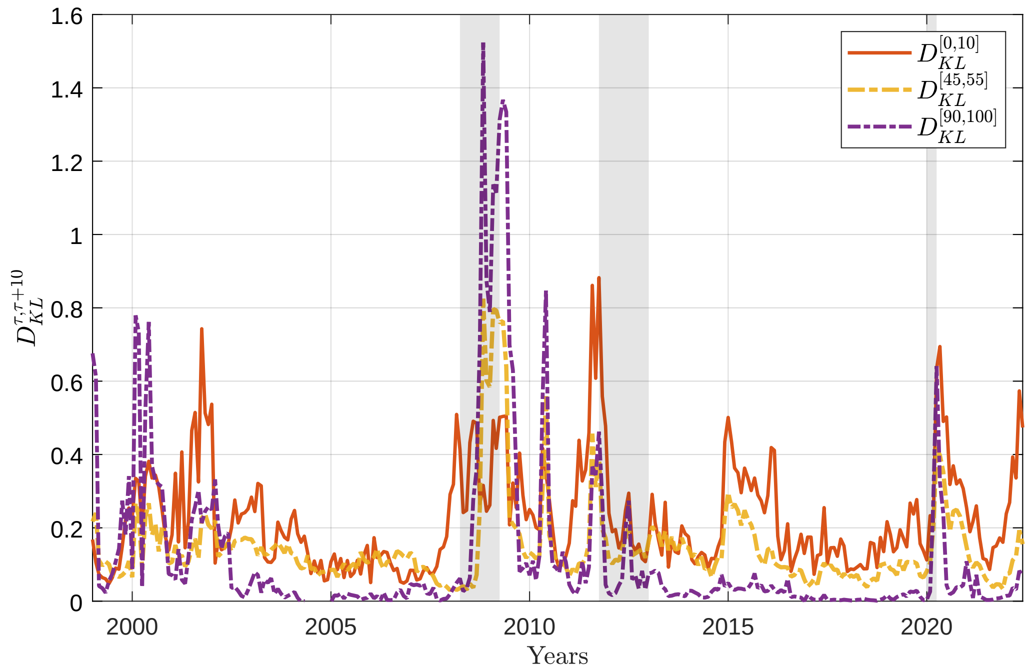
and

$$\hat{Q}_{\tau,-j}(\bar{\pi}^i_{t+1,t+h}|x^i_t) = x^i_{t,-j}\hat{\beta}^i_{\tau,-j}, \tag{8}$$

where j stands for the $j-$element and $(-j)$ for the exclusion of the $j-$element from the set of J variables. Therefore, the first term in the right-hand side of equation (7) measures the contribution of variable j to the quantile of future inflation and the second one the contribution of the other variables, defined by equation (8).

Using this decomposition, we compare KL measures $D_{KL,t,-j}(h)$ based on $\hat{Q}_{\tau,-j}(\bar{\pi}^i_{t+1,t+h}|x^i_t)$, that is the quantile predicted without variable j, to our benchmark measure $D_{KL,t}(h)$, based on $\hat{Q}_\tau(\bar{\pi}^i_{t+1,t+h}|x^i_t)$, when all variables are taken into account. $D_{KL,t,-j}(h) < D_{KL,t}(h)$ means

FIGURE 7. The Risk of Inflation Dispersion by Quantile



Note: The figure shows the evolution of KL divergence $D_{KL,t}^{[\tau,\tau+10]}(h=12)$ for quantiles $\tau = 0$, 45, and 90 defined by equation (5) over the sample period. Gray shaded areas indicate CEPR-dated recessions.

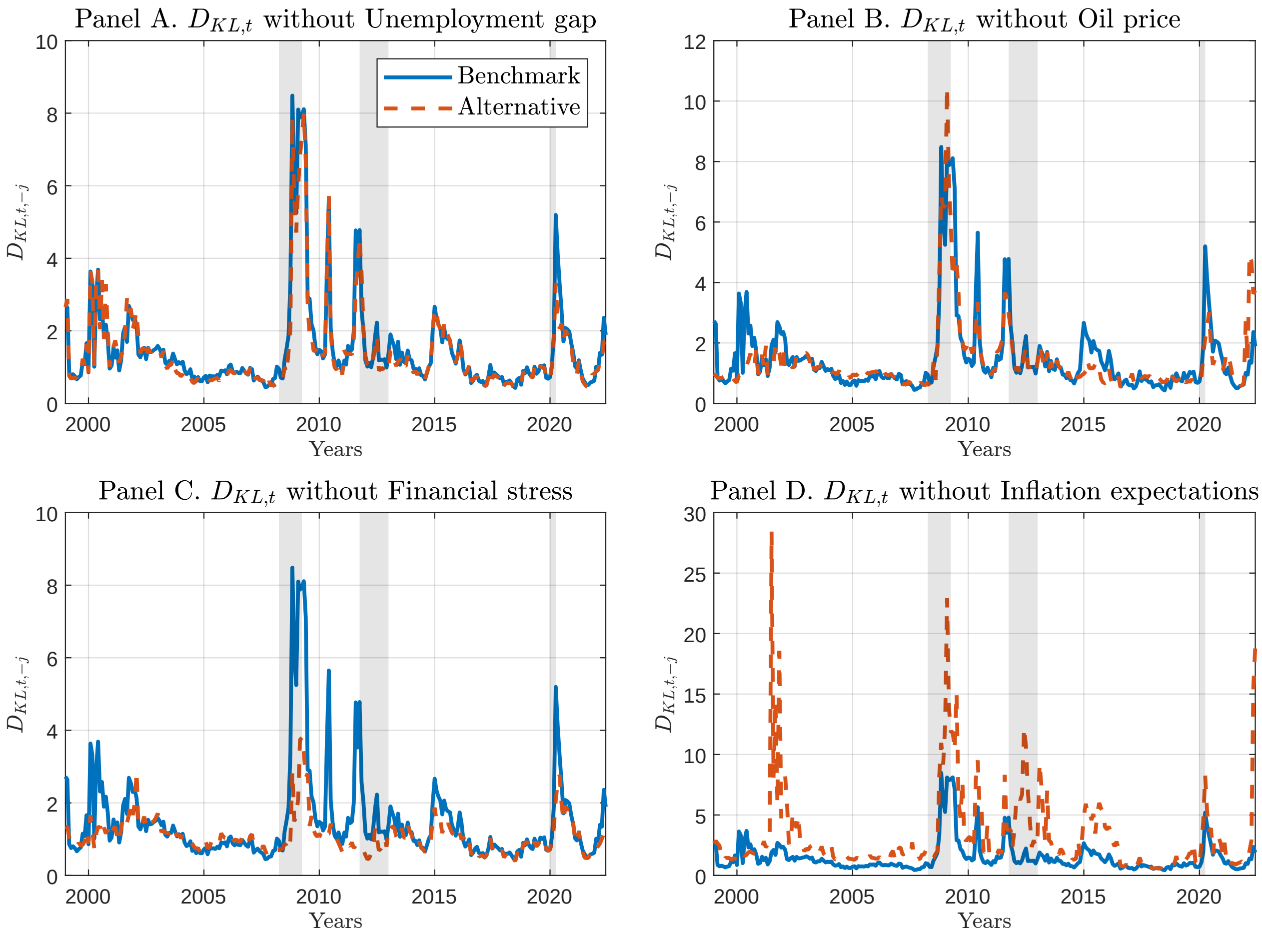
that the driver j is a source of divergence since the KL is lower when the driver j is not taken into when measuring divergence. Conversely, if $D_{KL,t,-j}(h) > D_{KL,t}(h)$, the driver j is then a source of convergence since the KL is higher when the driver j is not taken into account when measuring divergence. For comparison purposes, alternative KL statistics (i.e., KL measure without variable j) are computed only for $h = 12$.

Figure 8 compares the KL statistics $D_{KL,t,-j}$ for four inflation drivers.[17] Panel A shows that unemployment plays a minor role inflation divergence. The two lines are very close, except during the COVID crisis when removing this driver reduces KL from 5.19 to 3.32. Apart this episode, unemployment gap turns out to be a minor source of risk of inflation dispersion in the euro area (the mean of the KL is 1.48 in the benchmark case, slightly higher that of 1.39 when unemployment gap is removed).

Panel B shows that the gap between KL statistics is more substantial when oil price is removed instead of unemployment gap. Indeed, several peaks in inflation divergence for the benchmark (the solid blue line) are not observed for the alternative KL (the dashed red line) when oil price is removed from the inflation drivers. This is the case in the early 2000s and during the COVID crisis, but also in and 2010, 2011 and 2015. Interestingly, we observe

[17]So, four of the six variables considered in the Phillips curve. We do not consider the KL when the constant and past inflation are removed.

Figure 8. The Risk of Inflation Dispersion by Inflation Driver



Note: The figure compares the benchmark KL statistic $D_{KL,t}(h = 12)$ to the alternative KL statistics $D_{KL,t,-j}(h = 12)$ when inflation driver j is removed for j=[Unemployment gap, Oil price, Financial stress, Inflation expectations]. Gray shaded areas indicate CEPR-dated recessions.

the opposite for the last years and in 2009: the KL statistic is higher when oil price is not taken into account. So even if in average, oil price is minor source of inflation dispersion risk (the mean of KL without oil price is equal to 1.38, very close to that of computed when unemployment gap is removed) but can be punctually a source of divergence or convergence of inflation in the euro area.

Panel C shows that financial stress is a key source of inflation risk in the euro area. The mean of the KL without financial indicator is 1.08, e.g. around 30% lower than for the benchmark. The KL is substantially lower during the period 2008-2015 of financial turbulence (the dashed red line is below the solid blue one), but also in the early 2000s and during the COVID crisis. We further explore the role of financial conditions in the divergence of predictive inflation distribution among countries in the online Appendix and find evidence that financial stress is a key feature of dispersion in the tails of the distribution.

Finally, Panel D shows that inflation expectations are a key source of inflation convergence in the euro area. When inflation expectation is removed from the inflation drivers, the KL statistic skyrockets several times to values above 20. This demonstrates the importance of anchoring expectations, which makes converge inflation forecasts between countries and thus limits the risk of inflation dispersion in the euro area.

IV.3. **KL divergence across countries.** To conclude this section on the characteristic of the risk of inflation dispersion, we assess the role of each country in the divergence of predictive distributions. We recompute the KL measure for eleven countries by sequentially removing each of the twelve countries. In practical terms, we compute the divergence, $D_{KL,t,-k}(h)$, at horizon h without country k as

$$D_{KL,t,-k}(h) = \frac{1}{(N-1)[(N-1)-1]} \sum_{i}^{N-1} \sum_{j}^{N-1} KL_{i,j,t}(h)$$

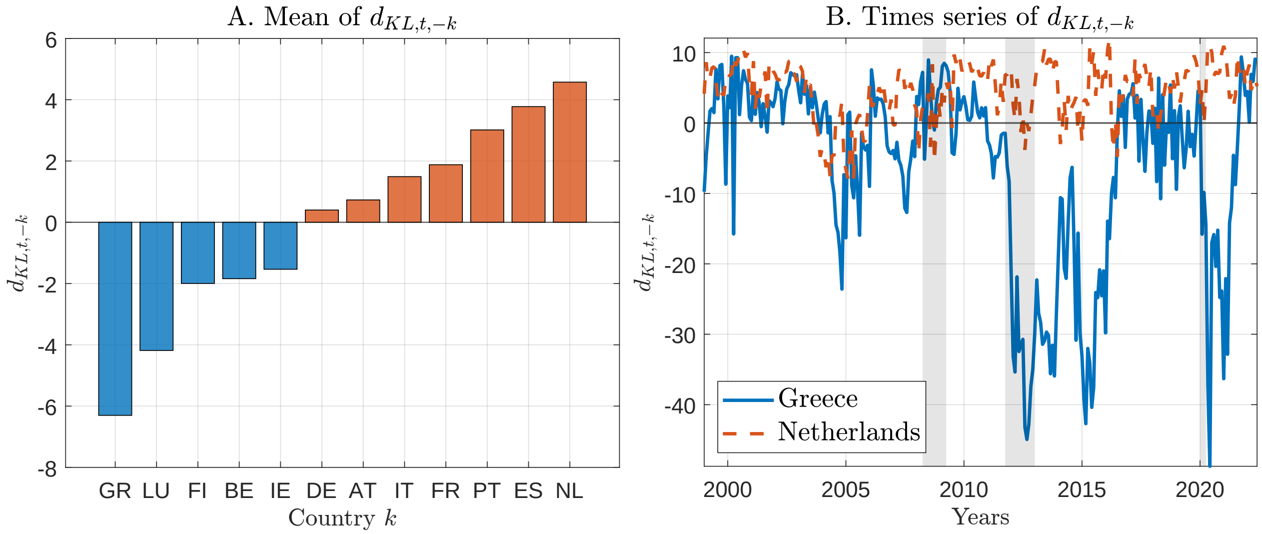
for $i \neq j$, $i \neq k$, $j \neq k$ and $k = 1, ..., N$, where $KL_{i,j,t}(h)$ is still defined by equation (4). The difference with respect to $D_{KL,t}(h)$, defined by equation (3), is that the k-country is not considered to compute $D_{KL,t,-k}(h)$ which therefore measures the divergence between all countries apart k. To measure the role of country k in the divergence of inflation risks, we compute the deviation in percentage between the two KL statistics as follows:

$$d_{KL,t,-k}(h) = 100 \times \frac{D_{KL,t,-k}(h) - D_{KL,t}(h)}{D_{KL,t}(h)}. \tag{9}$$

If $d_{KL,t,-k}(h)$ is negative, it means that the country k is a source of divergence to the extent that the KL is lower without the country k than when this country is included to compute the KL. Conversely, a positive $d_{KL,t,-k}(h)$ means that the country k is a source of convergence to the extent that the KL is higher without country k than when this country is included to compute the KL. As in previous exercises, we set $h = 12$.

Panel A of Figure 9 reports the mean values of $d_{KL,t,-k}$ for each country k over the sample period. Five countries are source of divergence of inflation risks (Greece, Luxembourg, Finland, Belgium, Ireland) and seven source of convergence (Germany, Austria, Italy, France, Portugal, Spain, Netherlands). It is important to note that our proposed measure of inflation risk dispersion is not determined by a single marginal country. If we consider the two extreme cases, KL variations are relatively modest: removing Greece from the sample reduces dispersion by 6.3%, while excluding Netherlands increases it by 4.5%. No single country is the only source of dispersion in the euro area on average over the period. This result does not mean that there are not periods when certain countries play a dominant role in the risk of inflation dispersion, especially during financial crisis.

Figure 9. The Risk of Inflation Dispersion by Country



Note: Panel A reports the mean values of $d_{KL,t,-k}(h = 12)$, defined by equation (9), for each country k over the sample period: $\bar{d}_{KL,-k}(h = 12) = (1/T) \times \sum_{t=1}^{T} d_{KL,t,-k}(h = 12)$. X-axis indicates country k. Countries are ranked in ascending order. Panel B shows the values of $d_{KL,t,-k}(h = 12)$, defined by equation (9) for countries k=[Greece, Netherlands] over the sample period. Gray shaded areas indicate CEPR-dated recessions.

Panel B of Figure 9 reports the values of $D_{KL,t,-k}$ for the two extreme countries, namely Greece and Netherlands, over the sample period (Figure E4 in the online Appendix reports the values for all countries). They have radically different patterns. Without Netherlands, the KL measure would have been slightly higher for all the period considered without large differences. On the contrary, Greece has been at the origin of a huge increase in the risk of inflation dispersion during the sovereign debt crisis. The KL between 2012 and 2015 would have been drastically reduced up to 45% in September 2012.

V. Policy Implications

This sections discusses the policy implications of our measure of expected inflation dispersion with regard to inflation forecasting and monetary policy decisions.

V.1. **The impact of risk of inflation dispersion on inflation realizations.** This section examines whether the information present in our expected inflation differential measures is useful for predicting future inflation in the euro area. We investigate predictive regressions in the context of final statistical releases. The variables we are interested in forecasting at horizon H, y_{t+H}, are HICP inflation and HICP inflation excluding food and energy.

To explore whether inflation differential risk provides insights into future inflation outcomes, we employ established and extensive modeling techniques, wherein predictions rely on estimations of common dynamic factors. Initial applications of dynamic factor models

(DFMs) to macroeconomic data indicated that a limited set of factors can explain a significant portion of the observed fluctuations in key economic indicators (e.g., Sargent and Sims, 1977; Stock and Watson, 1989, 1991; Sargent, 1989). By following this long tradition, our forecasting exercises test whether our variables of interest have a forecast power beyond the information content of typical inflation forecasts.

All models are specified and estimated as a linear projection of a H-step-ahead variable, y^H_{t+H}, onto t-dated predictors. Specifically, the baseline forecasting models all have the form

$$y^H_{t+H} = \mu + \sum_{i=1}^{p} \alpha_i y_{t-i+1} + \sum_{j=1}^{k} \beta_j' F_{t-j+1} + \gamma D_{KL,t}(h) + \varepsilon^H_{t+H},$$

where μ is a constant, α_i, β_j and γ are unknown parameters, $D_{KL,t}(h)$ contains our measure of inflation divergence at horizon h, and F_t is a $r \times 1$ vector of predictor variables, which is set to be the principal components from a large number of candidate predictor time series, $Z_t = (Z_{1t}, ..., Z_{Nt})$,

$$Z_t = \Lambda F_t + e_t, \qquad t = 1, ..., T.$$

We consider a set of $N = 39$ variables in Z_t that are related to macroeconomic, survey and financial time series. The series are transformed by taking logarithms and/or differencing. In general, first differences of logarithms (growth rates) are used for real quantity variables, and first differences are used for nominal interest rates. The list of series and transformations are reported in Table F4 in the online Appendix. We then extract their principal components using the factor extraction technique developed by Bai and Ng (2002). The number of estimated factors is equal to 5.

Our focus is on multistep-ahead prediction, and most of the forecasting regressions are projections of an H-step-ahead variable y^H_{t+H} onto t-dated predictors, including also lagged transformed values y_t of the variable of interest. The price indexes are modeled as being I(1) in logarithms. For example, when forecasting the monthly HICP:

$$y_{t+H} = (1200/H)\ln(HICP_{t+H}/HICP_t), \qquad \text{and} \qquad y_t = 1200\ln(HICP_t/HICP_{t-1}),$$

the H-period growth rate expressed in percentage points at an annual rate. Forecasts are made at forecast horizon of one year and two years (twelve and twenty-four months, respectively).

The results are reported in Table III for both HICP (left panel) and HICP excluding food and energy (right panel) at twelve- and twenty-four-months-ahead forecasts. The number of lags of y_t, is chosen by Bayesian information criterion (BIC), with $0 \leq p, k \leq 6$. For brevity, we omit the constant, coefficients of factors, and lagged dependent variables.

Table III. The effect of Risk of Inflation Dispersion on Inflation Realizations

	HICP		HICP excl. food and energy	
	12-months	24-months	12-months	24-months
	(1)	(2)	(3)	(4)
$D_{KL,t}(h=3)$	0.133	0.244**	0.042	0.136***
	(0.120)	(0.099)	(0.041)	(0.049)
Observations	246	246	246	246
Adjusted R^2	0.19	0.23	0.45	0.37
$D_{KL,t}(h=6)$	0.432***	0.446***	0.128***	0.257***
	(0.089)	(0.077)	(0.042)	(0.036)
Observations	246	246	246	246
Adjusted R^2	0.25	0.32	0.48	0.48
$D_{KL,t}(h=9)$	0.395***	0.338***	0.075***	0.125***
	(0.064)	(0.056)	(0.028)	(0.034)
Observations	246	246	246	246
Adjusted R^2	0.28	0.32	0.47	0.40
$D_{KL,t}(h=12)$	0.219***	0.224***	0.052**	0.113***
	(0.046)	(0.039)	(0.021)	(0.024)
Observations	246	246	246	246
Adjusted R^2	0.23	0.28	0.46	0.41
$D_{KL,t}(h=24)$	0.279***	0.329***	0.011	0.123***
	(0.069)	(0.062)	(0.033)	(0.035)
Observations	246	246	246	246
Adjusted R^2	0.22	0.28	0.45	0.38

Note: Statistical significance is shown for * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parentheses are robust to heteroskedasticity. For brevity, the table omits constant, coefficients of factors, and lagged dependent variables. OLS estimation of equation (V.1).

For the considered forecasting horizons, and after accounting for the macroeconomic and financial controls contained in F_t, the risk of inflation divergence, $D_{KL,t}(h)$, has a significant impact on future inflation realizations for HICP at one- and two-years horizons, as indicated in columns (1)-(2). Results are highly significant as shown by standard errors given in parentheses (the single exception is for the 1-year forecast when $h = 3$). According to our baseline measure, $D_{KL,t}(h = 12)$, a one standard deviation increase in the risk of inflation dispersion predicts a $0.22 \times \text{std}\left(D_{KL,t}(h = 12)\right) = 0.3$ percentage points increase in inflation at horizons of twelve and twenty-four months. Examining HICP excluding food and energy (columns (3)-(4)), our results are still significant.

We complement this initial exercise with an assessment of the forecasting efficacy of our expected inflation differential measures. Using the average of forecast errors, we compare the inflation forecasting model employing factors and lagged endogenous variables with one that additionally incorporates measures of inflation divergence. We will denote the ratio of mean-squared errors as RMSE (relative mean-squared error). A value less than one signifies that the model incorporating our inflation differential measures is superior.

Table IV presents the results associated with both forecasting horizons for $D_{KL,t}(h = 12)$. The findings are striking: for any horizon, our measure of divergence improves the precision of in-sample forecasts for both inflation measures by 9% to 13%, with p-values less than 0.05, indicating rejection of the null hypothesis of equal accuracy between the two models at the 5% significance level. Although not reported here, results are quantitatively similar for other horizons of D_{KL}.

Table IV. Forecast Performances

	HICP		HICP excl. food and energy	
	12-months (1)	24-months (2)	12-months (3)	24-months (4)
$D_{KL,t}(h = 12)$	0.9231	0.8943	0.9103	0.8701
	(0.0185)	(0.0021)	(0.0210)	(0.0168)

Note: Entries are RMSE ratios associated to inflation forecasts. Each panel compares a forecasting model of inflation using the factors and lagged endogenous variable (model 1) with one where $D_{KL,t}(h = 12)$ is also added (model 2). An entry less than one indicates that model 2 is superior to model 1. Values in parentheses report the p-values of the Diebold and Mariano (1995)-West (1996) test statistic for equal predictive accuracy.

V.2. **The risk of risk of inflation dispersion and ECB monetary policy.** The 2003 strategy review and recent speeches by central bankers confirm the view that the ECB may have paid attention to euro area inflation differentials over the past decades. The purpose of this section is then to check the extent to which our measures of inflation dispersion matter for ECB's interest rate setting decisions.

To assess the extent to which the euro area policy rate reacts to the risk of inflation dispersion, we estimate a first-difference forward-looking monetary policy rule as originally proposed by Orphanides (2003) to describe U.S. Federal Reserve during the Volcker-Greenspan era. This specification of the reaction function has been applied to describe euro area policy rates by Orphanides and Wieland (2013), Bletzinger and Wieland (2017), and Hartmann and Smets (2018), among others. This monetary policy rule offers two main advantages. First, it avoids the use of unobservable concepts such as the output gap or the natural interest rate in the specification, which are subject to considerable uncertainty and data revisions. Second, it offers a powerful real-time policy benchmark, based on forecasts for inflation and output growth that were available at the time of monetary policy decisions.

Following the methodology employed in the above-mentioned papers, we first estimate the following baseline specification of the euro area monetary policy rule:

$$\Delta i_t = \alpha + \beta(E_t\pi_{t+1} - \pi^*) + \gamma(E_t\Delta y_{t+1} - \Delta\bar{y}_t) + \varepsilon_t, \tag{10}$$

where Δi_t is the quarter-on-quarter change in the key policy rate, $E_t\pi_{t+1} - \pi^*$ is the one-year-ahead expected inflation rate in deviation from ECB's inflation target, and $E_t\Delta y_{t+1} - \Delta\bar{y}_t$ is the deviation of one-year-ahead expected real GDP growth from the potential output growth. We then augment the forward-looking policy rule given in equation (10) with our measure of dispersion risk in inflation at horizon h:

$$\Delta i_t = \alpha + \beta(E_t\pi_{t+1} - \pi^*) + \gamma(E_t\Delta y_{t+1} - \Delta\bar{y}_t) + \theta_h D_{KL,t}(h) + \varepsilon_t,$$

To be in line with the rest of the paper, our dispersion measures $D_{KL,t}(h)$ covers the KL metrics at different horizon h developed in Section III. Table V reports estimated coefficients for the simple first-difference policy rule, and the outcome of augmented reaction function estimation, which alternatively includes our measures of KL divergence. The results of the simple first-difference monetary policy rule estimates are consistent with those discussed in the literature (e.g., Hartmann and Smets, 2018). The response coefficients on inflation and growth forecasts are all positive and significant at 1%. Moreover, the adjusted R^2 is higher than 50%, meaning that macroeconomic forecasts from the SPF have a strong explanatory power of ECB's key policy rate decisions, given the fact that the dependent variable is expressed in first-differences.

Augmenting the first-difference monetary policy rule with our KL metrics does not considerably change the response coefficients of inflation and growth forecasts, regarding both the magnitude and the statistical significance. Regarding the interest rate reaction to KL divergence (θ_h), its estimates is negative across all horizons, but appears to be significant only at shorter horizons up to one year. A one standard deviation increase in the measure of inflation dispersion is found to decrease the target interest rate by 6 to 13 basis points. Note also that the magnitude of the response coefficient is decreasing as long as the time horizon of the KL divergence is increasing at shorter horizons, despite being relatively high for $h \geq 21$. Overall, our results suggest that the risk of inflation dispersion has predictive power for the ECB's policy rate. These results may also be interpreted as ECB's concerns about the risk of euro area financial fragmentation: the ECB tends to lower the policy rate in response to a high risk of inflation dispersion which could potentially lead to a dispersion of real interest rates.

Table V. Regression Results for the Orphanides Rule

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
SPF inflation projections	0.23***	0.21***	0.25***	0.22***	0.21***	0.21***	0.21***	0.21***	0.20***
	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)	(0.07)
SPF growth projections	0.34***	0.29***	0.28***	0.31***	0.26***	0.27***	0.30***	0.29***	0.28***
	(0.08)	(0.06)	(0.05)	(0.06)	(0.05)	(0.05)	(0.06)	(0.06)	(0.06)
$D_{KL,t}(h=3)$		-0.13***							
		(0.04)							
$D_{KL,t}(h=6)$			-0.08**						
			(0.03)						
$D_{KL,t}(h=9)$				-0.06**					
				(0.03)					
$D_{KL,t}(h=12)$					-0.06**				
					(0.03)				
$D_{KL,t}(h=15)$						-0.05			
						(0.03)			
$D_{KL,t}(h=18)$							-0.03		
							(0.03)		
$D_{KL,t}(h=21)$								-0.06	
								(0.04)	
$D_{KL,t}(h=24)$									-0.08*
									(0.04)
Observations	84	84	84	84	84	84	84	84	84
Adjusted R^2	0.55	0.61	0.59	0.59	0.60	0.58	0.56	0.57	0.59

Note: Statistical significance is shown for * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. Standard errors in parentheses are robust to heteroskedasticity. Estimation results for the constant are not reported. Column (1) shows OLS estimation results for equation (10). Columns (2) to (9) show OLS estimation results for equation (V.2). The sample period is 1999:Q1 to 2019:Q4. The dependent variable is measured as changes in the key ECB policy rate, which combines the time series of the changes in the main refinancing operations (MRO) rate up to 2008:Q3 with the changes in the deposit facility rate (DFR) from 2008:Q4 onward. Changes are mid-quarter-on-quarter changes. The variables SPF inflation projections and SPF growth projections refer to one-year-ahead HICP projections and one-year-ahead real GDP growth projections in deviation from potential growth of the ECB Survey of Profesionnal Forecasters, respectively. Potential growth is measured as the annual growth rate of European Commission potential GDP. KL divergence series are mid-quarter levels over the sample period.

VI. Conclusion

We introduced a comprehensive methodology for measuring the risk of inflation dispersion among euro area countries over time. The approach considered the degree of dissimilarity in predictive inflation distributions among euro area countries. By doing so, it addressed not only cross-countries differences in point forecasts of inflation, but also cross-countries differences in uncertainty and tail risks. Based on our measure, we documented that the rising risk of inflation dispersion is mainly driven by a deterioration in financial conditions, while a robust anchoring of inflation expectations in each country tends to mitigate this risk. Finally, we showed the risk of inflation dispersion possesses predictive efficacy for future inflation realizations as well as variations in the monetary authority's interest rate.

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ONLINE APPENDIX: THE RISK OF INFLATION DISPERSION IN THE EURO AREA

STÉPHANE LHUISSIER, AYMERIC ORTMANS AND FABIEN TRIPIER

This Appendix consists of the following sections:

Appendix A. Data

All variables are monthly time series covering January 1999 through July 2023. The following variables use data obtained directly from different sources:

- Harmonized Index of Consumer Prices
  - Source: ECB - ICP (Indices of Consumer prices)
  - Details: Monthly – Neither seasonally nor working day adjusted – HICP - All-items excluding energy and food – Eurostat – Index
  - Data transformation: Authors' calculations using the x13 toolbox to get seasonally adjusted series for each euro area member countries.
- Unemployment rate
  - Source: Eurostat - Unemployment by sex and age – monthly data
  - Details: Monthly – Seasonally adjusted data, not calendar adjusted data – Total – Percentage of population in the labor force
- Natural Rate of Unemployment
  - Source: Authors' calculations
  - Details: HP-filtered trend (with smoothing parameter $\lambda = 14,400$ of unemployment rate).
- Oil Prices
  - Source: U.S. Energy Information Administration - Spot Prices
  - Details: Crude Oil Prices: Brent - Europe - Dollars per Barrel, Not Seasonally Adjusted
- Financial conditions (CISS)
  - Source: ECB - CISS
  - Details: Daily – ECB – Economic indicator – New Composite Indicator of Systemic Stress (CISS) – Index
  - Data transformation: Authors' calculations to get monthly average of the series.
- Financial conditions (CLIFS)
  - Source: ECB - CLIFS
  - Details: Monthly – ECB – Economic indicator – Country-Level Index of Financial Stress (CLIFS) Composite Indicator – Index
- Long-Term Inflation Expectations
  - Source: Consensus Economics
  - Details: Six-to-ten-years-ahead mean CPI inflation forecasts.
  - Data transformation: Euro area forecasts for Luxembourg (no forecast available), spline interpolation for all missing data in April 1999.

Appendix B. National Phillips Curve Estimates (tables)

This section presents the results of the quantile Phillips curve estimates by country. The results are displayed in Tables B1 to B3. Each table reports the estimated coefficients of equation (2) for each country for quantiles $\tau = \{10, 50, 90\}$, respectively. The last two rows of the tables report the unweighted means and the standard deviations of coefficients across countries.

First of all, the mean and the standard deviation of coefficient λ_τ^i associated to long-term inflation expectations across countries remain relatively stable over the 10^{th} and the 50^{th} quantiles (around 0.55 and 0.25, respectively). However, the mean of the coefficient becomes lower at the top of the distribution (90^{th}) at around 0.39. Overall, the average weight of inflation expectations is greater than that of past inflation for the 10^{th} and 50^{th} quantiles, but is lower at the top of the distribution. This result is in line with Baba et al. (2023) who also find that inflation has become increasingly backward looking across Europe since the COVID pandemic. Our results also reveal that inflation anchoring is not the same when looking at the weight of inflation expectations country-by-country. For instance, the coefficient is close to 1 in Germany in the middle of the distribution (50^{th} quantile), whereas it is close to zero at the top of the distribution (90^{th} quantile). Inversely, the coefficient is equal to 0.49 in Greece in the middle of the distribution but increases to 0.84 at the top of the distribution.

Focusing on the θ_τ^i coefficient (i.e., the slope of the Phillips curve), the magnitude of the cross-sectional mean is twice higher for the 50^{th} and 90^{th} quantiles than for the 10^{th} quantile, though the coefficient is generally not significant from zero. Unemployment seems to affect inflation much more in the middle or at the top of the distribution than at the bottom in the euro area, on average. This result suggests that labor market conditions matter more for upside risks to inflation than for downside inflation risks. Such nonlinearities in the relationship between slack and inflation corroborate those from Gagnon and Collins (2019) in which the Phillips curve is normally steep but becomes nonlinear only when inflation is low. Once again, even if the cross-sectional standard deviation does not change significantly from a quantile to another, the estimated slope of the Phillips curve shows important disparities across countries within and between quantiles. For instance, the coefficient is strongly negative in the Netherlands for the 10^{th} as for the 50^{th} quantile, but is close to zero at the top of the distribution. This highlights important disparities across countries for each quantile.

The cross-sectional mean of the coefficient associated with financial stress, δ_τ^i, is negative and higer at the tails (10^{th} and 90^{th} quantiles) than at the middle of the distribution (-1.19 and -1.34 against -0.78). Although surprising at the top of the distribution, this result is consistent with the role of tighter financial conditions in the occurrence of low inflation episodes in the euro area.[18] Our results corroborate a vast literature maintaining that there is a nonlinear relationship between financial sector and macroeconomy depending on the state of the economy. Notable examples include He and Krishnamurthy (2012, 2013) and Brunnermeier and Sannikov (2014) for the theory, and Hubrich and Tetlow (2015) and Lhuissier (2017) for the empirics. Since this coefficient is the only one to be left unconstrained in the benchmark specification of the augmented Phillips curve model, it shows important disparities between euro area countries. The cross-sectional standard deviation is indeed very high for the three quantiles (1.48 for the 50$^{\text{th}}$ quantile, 1.58 for the 90$^{\text{th}}$ quantile, and 2.37 for the 10$^{\text{th}}$ quantile). However, as for the other estimated coefficients of the model, the effect of financial stress on inflation varies across countries and over the quantiles. For instance, the coefficient is positive at the top but negative at the bottom of the distribution in Austria (0.35 for the 90$^{\text{th}}$ quantile and -0.21 for the 10$^{\text{th}}$ quantile), whereas it is higher (but always negative) in Greece at the bottom of the distribution (-5.67 for the 10$^{\text{th}}$ quantile and -3.06 for the 90$^{\text{th}}$ quantile).

Finally, the cross-sectional mean of the γ_τ^i coefficient is much higher for the 90$^{\text{th}}$ than for the 10$^{\text{th}}$ quantile, suggesting that oil price affects upside risks to inflation relatively more than downside inflation risks (1.63 against 0.60).

As a whole, and despite constrained coefficients (except on financial conditions), estimated national Phillips curve results show important non-linearities across quantiles. Moreover, it is worth noting that the non-linearities across quantiles are not the same for all countries, providing grounds for looking at the dispersion of conditional quantiles across euro area countries.

[18] López-Salido and Loria (forthcoming) also find that the tails of euro area inflation predictive distribution are equally negatively affected by tighter financial conditions, contrasting with their main finding using U.S. data.

Table B1. Phillips Curve Estimates for the 10^{th} Quantile

	$\hat{\mu}_\tau^i$	$\hat{\lambda}_\tau^i$	$\hat{\theta}_\tau^i$	$\hat{\gamma}_\tau^i$	$\hat{\delta}_\tau^i$
Germany	−1.12 [−1.90;−0.34]	0.47 [0.33;0.61]	−0.00 [−0.13;0.13]	0.38 [−0.42;1.19]	0.54 [−1.53;2.61]
France	−0.72 [−1.50;0.05]	0.27 [0.13;0.41]	−0.10 [−0.23;0.03]	0.47 [−0.32;1.27]	−0.44 [−2.51;1.64]
Italy	−0.59 [−1.37;0.19]	0.19 [0.05;0.33]	−0.01 [−0.14;0.12]	0.16 [−0.64;0.96]	−1.22 [−3.29;0.85]
Spain	−0.97 [−1.74;−0.19]	0.43 [0.29;0.57]	−0.00 [−0.13;0.13]	0.91 [0.10;1.71]	−4.55 [−6.60;−2.51]
Netherlands	−1.29 [−2.08;−0.51]	0.95 [0.81;1.09]	−0.69 [−0.81;−0.56]	0.64 [−0.17;1.44]	0.36 [−1.70;2.42]
Finland	−1.14 [−1.91;−0.37]	0.54 [0.40;0.68]	−0.02 [−0.15;0.11]	0.27 [−0.53;1.06]	1.09 [−0.97;3.15]
Ireland	−1.25 [−2.02;−0.48]	0.70 [0.55;0.84]	−0.00 [−0.13;0.13]	0.90 [0.09;1.70]	−4.57 [−6.61;−2.53]
Austria	−0.56 [−1.34;0.22]	0.74 [0.60;0.89]	−0.00 [−0.13;0.13]	0.00 [−0.80;0.80]	−0.21 [−2.29;1.86]
Portugal	−1.31 [−2.09;−0.53]	0.51 [0.37;0.65]	−0.00 [−0.13;0.13]	0.60 [−0.20;1.40]	−0.52 [−2.59;1.55]
Belgium	−0.75 [−1.51;0.02]	0.92 [0.77;1.06]	−0.13 [−0.26;−0.00]	0.30 [−0.50;1.09]	−0.12 [−2.13;1.89]
Luxembourg	−0.73 [−1.50;0.05]	0.39 [0.25;0.54]	−0.11 [−0.24;0.02]	0.72 [−0.09;1.53]	1.06 [−1.02;3.14]
Greece	−1.13 [−1.90;−0.36]	0.47 [0.33;0.61]	−0.03 [−0.16;0.10]	1.83 [1.03;2.63]	−5.67 [−7.70;−3.63]
Mean	-0.96	0.55	-0.09	0.60	-1.19
Std. Dev.	0.28	0.24	0.19	0.48	2.37

Note: Coefficients of the quantile Phillips curve defined by equation (2): $\hat{Q}_\tau(\bar{\pi}^i_{t+1,t+h}|x^i_t) = \hat{\mu}^i_\tau + \left(1 - \hat{\lambda}^i_\tau\right)\pi^{*,i}_{t-1} + \hat{\lambda}^i_\tau \pi^{LTE,i}_t + \hat{\theta}^i_\tau \left(u^i_t - u^{*,i}_t\right) + \hat{\gamma}^i_\tau \left(\pi^{o,*}_t - \pi^{*,i}_t\right) + \hat{\delta}^i_\tau f^i_t$ estimated by country for the 10^{th} quantile. The last two rows show the unweighted means and the standard deviations of coefficients across countries. 68% confidence intervals are in brackets and are based on block-by-block bootstrap (10,000 draws) developed in Kilian and Kim (2011).

Table B2. Phillips Curve Estimates for the 50th Quantile

	$\hat{\mu}_\tau^i$	$\hat{\lambda}_\tau^i$	$\hat{\theta}_\tau^i$	$\hat{\gamma}_\tau^i$	$\hat{\delta}_\tau^i$
Germany	−0.49 [−0.84;−0.14]	0.95 [0.81;1.08]	−0.08 [−0.39;0.23]	0.37 [−0.25;0.98]	−0.32 [−3.30;2.67]
France	−0.11 [−0.46;0.24]	0.31 [0.18;0.45]	−0.01 [−0.31;0.30]	0.24 [−0.37;0.85]	−0.82 [−3.77;2.14]
Italy	−0.01 [−0.36;0.35]	0.20 [0.07;0.34]	−0.23 [−0.53;0.08]	0.13 [−0.49;0.74]	−0.78 [−3.75;2.19]
Spain	−0.15 [−0.50;0.19]	0.52 [0.39;0.66]	−0.00 [−0.31;0.31]	0.47 [−0.14;1.08]	−1.22 [−4.17;1.73]
Netherlands	−0.36 [−0.71;−0.01]	0.62 [0.48;0.75]	−1.03 [−1.33;−0.72]	0.93 [0.32;1.55]	0.15 [−2.79;3.08]
Finland	−0.23 [−0.57;0.12]	0.24 [0.10;0.37]	−0.02 [−0.33;0.29]	0.99 [0.39;1.60]	0.54 [−2.41;3.49]
Ireland	−0.32 [−0.66;0.03]	0.37 [0.23;0.50]	−0.00 [−0.30;0.30]	1.47 [0.86;2.08]	−1.20 [−4.17;1.77]
Austria	−0.11 [−0.46;0.23]	0.80 [0.66;0.93]	−0.00 [−0.31;0.31]	0.35 [−0.26;0.96]	0.28 [−2.68;3.24]
Portugal	−0.23 [−0.58;0.12]	0.51 [0.38;0.65]	−0.00 [−0.31;0.31]	0.74 [0.13;1.34]	−1.44 [−4.38;1.50]
Belgium	−0.33 [−0.68;0.02]	0.72 [0.59;0.86]	−0.18 [−0.48;0.13]	0.53 [−0.07;1.14]	−0.26 [−3.23;2.72]
Luxembourg	−0.19 [−0.53;0.16]	0.87 [0.73;1.01]	−0.51 [−0.81;−0.20]	0.70 [0.09;1.31]	0.63 [−2.35;3.62]
Greece	0.21 [−0.13;0.56]	0.49 [0.35;0.62]	−0.33 [−0.64;−0.02]	0.83 [0.22;1.44]	−4.90 [−7.85;−1.95]
Mean	-0.19	0.55	-0.20	0.65	-0.78
Std. Dev.	0.18	0.25	0.31	0.38	1.48

Note: Coefficients of the quantile Phillips curve defined by equation (2): $\hat{Q}_\tau(\bar{\pi}^i_{t+1,t+h}|x^i_t) = \hat{\mu}^i_\tau + \left(1-\hat{\lambda}^i_\tau\right)\pi^{*,i}_{t-1} + \hat{\lambda}^i_\tau \pi^{LTE,i}_t + \hat{\theta}^i_\tau\left(u^i_t - u^{*,i}_t\right) + \hat{\gamma}^i_\tau\left(\pi^{o,*}_t - \pi^{*,i}_t\right) + \hat{\delta}^i_\tau f^i_t$ estimated by country for the 50th quantile. The last two rows show the unweighted means and the standard deviations of coefficients across countries. 68% confidence intervals are in brackets and are based on block-by-block bootstrap (10,000 draws) developed in Kilian and Kim (2011).

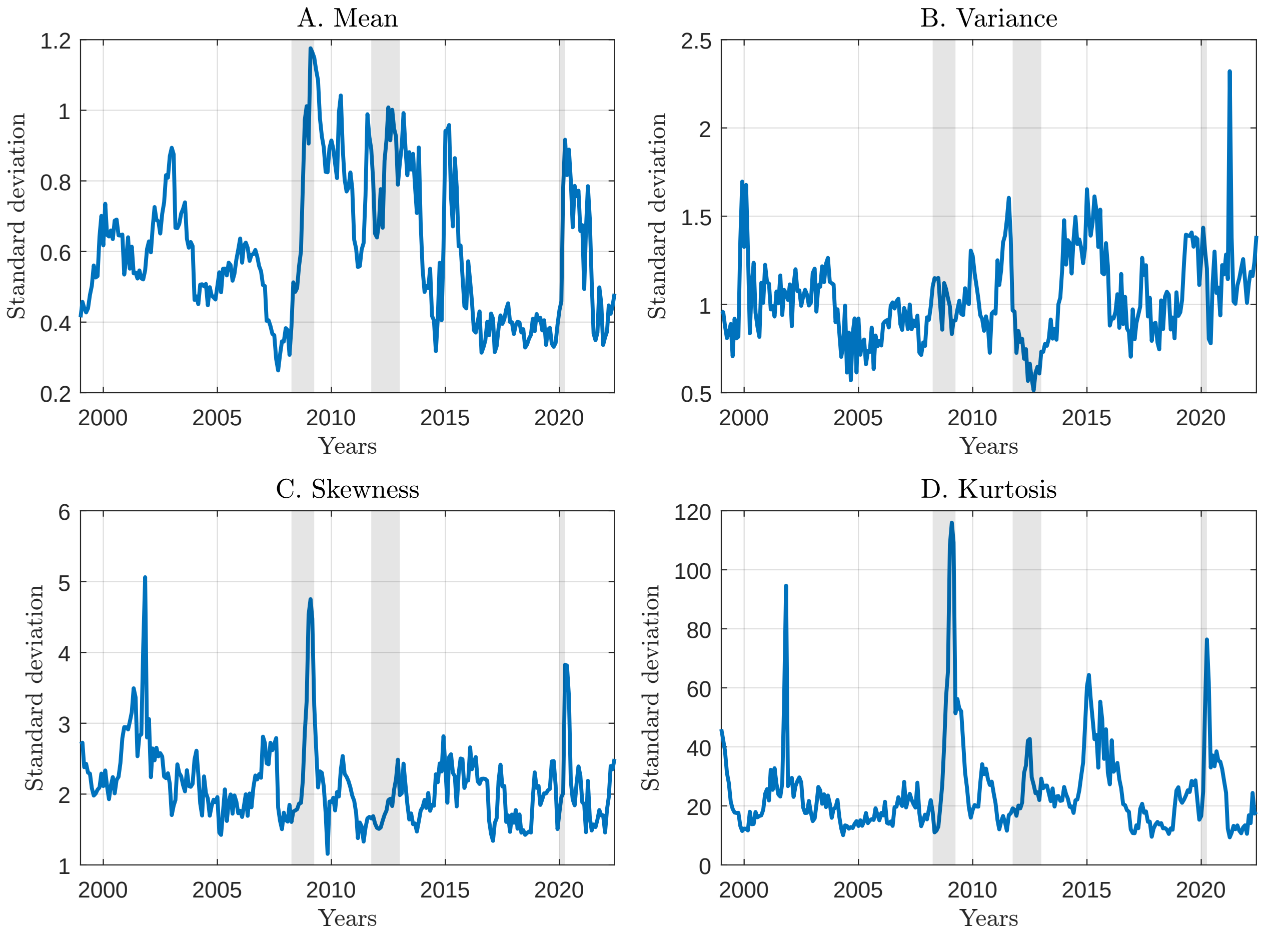
Table B3. Phillips Curve Estimates for the 90th Quantile

	$\hat{\mu}^i_\tau$	$\hat{\lambda}^i_\tau$	$\hat{\theta}^i_\tau$	$\hat{\gamma}^i_\tau$	$\hat{\delta}^i_\tau$
Germany	1.43 [0.88;1.98]	0.05 [−0.15;0.24]	−0.00 [−0.61;0.61]	0.72 [−0.00;1.44]	−1.64 [−4.55;1.27]
France	1.05 [0.50;1.60]	0.13 [−0.07;0.32]	−0.28 [−0.89;0.33]	0.90 [0.19;1.61]	−1.66 [−4.62;1.29]
Italy	1.20 [0.65;1.76]	0.34 [0.15;0.54]	−0.00 [−0.61;0.61]	1.38 [0.66;2.10]	−1.00 [−3.91;1.91]
Spain	1.17 [0.62;1.72]	0.70 [0.51;0.90]	−0.05 [−0.66;0.56]	1.23 [0.50;1.95]	−2.62 [−5.53;0.30]
Netherlands	1.26 [0.71;1.81]	0.16 [−0.04;0.36]	−0.17 [−0.78;0.44]	3.15 [2.43;3.86]	0.59 [−2.36;3.54]
Finland	0.52 [−0.02;1.07]	0.75 [0.56;0.95]	−0.00 [−0.61;0.61]	1.97 [1.24;2.69]	1.22 [−1.67;4.11]
Ireland	1.78 [1.23;2.32]	0.43 [0.23;0.62]	−0.00 [−0.61;0.61]	1.00 [0.28;1.72]	−3.09 [−6.00;−0.18]
Austria	1.14 [0.59;1.69]	0.00 [−0.20;0.20]	−0.00 [−0.61;0.61]	1.94 [1.23;2.66]	0.35 [−2.57;3.26]
Portugal	2.01 [1.47;2.56]	0.44 [0.24;0.63]	−0.23 [−0.84;0.38]	1.99 [1.27;2.71]	−3.57 [−6.46;−0.67]
Belgium	1.31 [0.76;1.86]	0.00 [−0.20;0.20]	−0.00 [−0.61;0.61]	1.83 [1.10;2.55]	−1.46 [−4.37;1.46]
Luxembourg	0.58 [0.03;1.13]	0.79 [0.60;0.99]	−0.00 [−0.61;0.61]	1.51 [0.79;2.22]	−0.17 [−3.07;2.74]
Greece	1.66 [1.10;2.22]	0.84 [0.64;1.03]	−1.11 [−1.72;−0.50]	1.92 [1.19;2.65]	−3.06 [−6.02;−0.10]
Mean	1.26	0.39	-0.15	1.63	-1.34
Std. Dev.	0.44	0.32	0.32	0.66	1.58

Note: Cefficients of the quantile Phillips curve defined by equation (2): $\hat{Q}_\tau(\bar{\pi}^i_{t+1,t+h}|x^i_t) = \hat{\mu}^i_\tau + \left(1-\hat{\lambda}^i_\tau\right)\pi^{*,i}_{t-1} + \hat{\lambda}^i_\tau \pi^{LTE,i}_t + \hat{\theta}^i_\tau\left(u^i_t - u^{*,i}_t\right) + \hat{\gamma}^i_\tau\left(\pi^{o,*}_t - \pi^{*,i}_t\right) + \hat{\delta}^i_\tau f^i_t$ estimated by country for the 90th quantile. The last two rows show the unweighted means and the standard deviations of coefficients across countries. 68% confidence intervals are in brackets and are based on block-by-block bootstrap (10,000 draws) developed in Kilian and Kim (2011).

Appendix C.

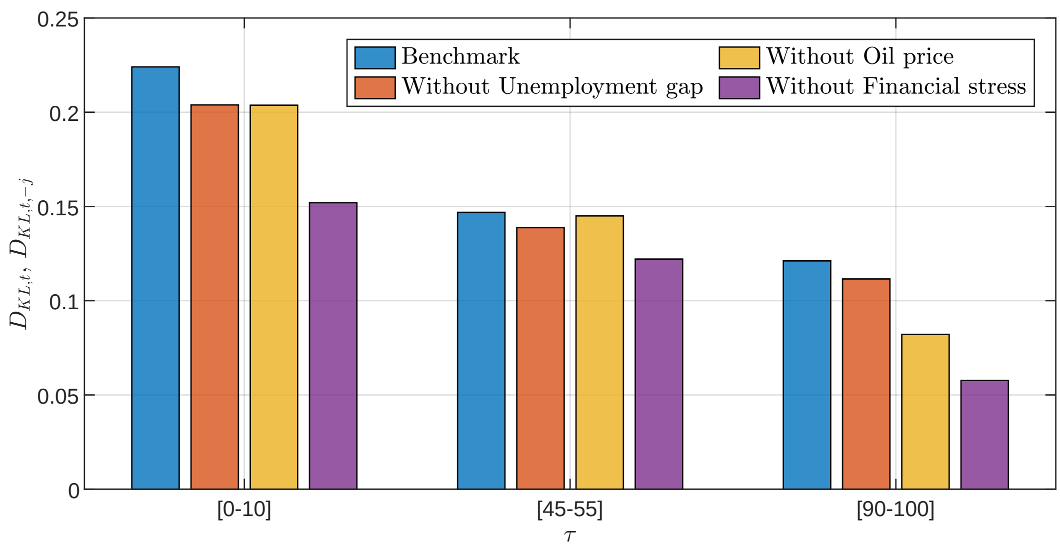
Figure C1. Cross-Country Dispersion of Skewed t-Distribution Moments over Time



Appendix D. The Role of Financial Conditions in KL divergence

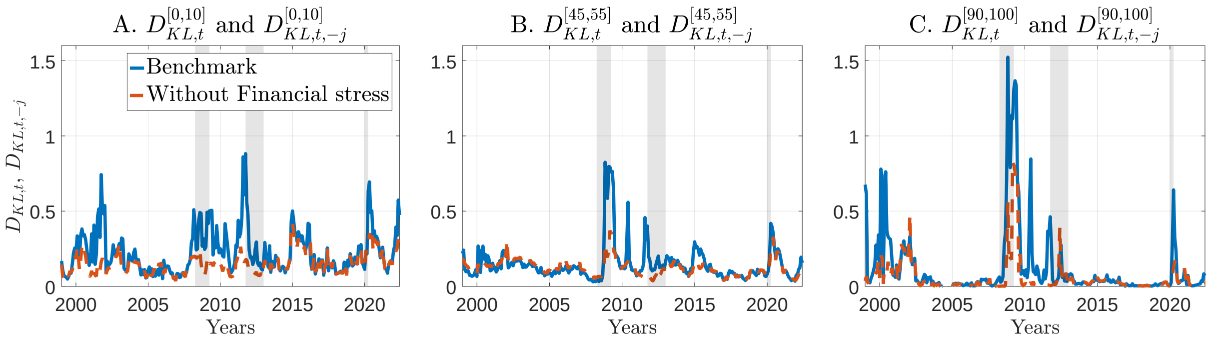
To further our understanding of the role of financial conditions, we can combine information by driver and quantile. We apply the quantile-based KL measure defined by equation (6) to predictive densities associated with quantiles $\hat{Q}_{\tau,-j}$ defined by equation (8). Figures D2 and D3 show that financial conditions is a key source of inflation dispersion risk at the tails of predictive distributions. Removing the financial stress indicator reduces by one third the average over the sample period of the KL associated with the quantiles $[0, 10]$ and by half that of associated with the quantiles $[90, 1000]$. Figure D2 shows that the left tails of predictive distributions are more dispersed than at the middle and at the right and that is comes mainly from the role of financial conditions. Panels A to C of Figure D3 confirm this conclusion by depicting a rather flat measure of the risk of inflation dispersion when financial stress indicator is muted.

Figure D2. The Risk of Inflation Dispersion by Inflation Driver and Quantile



Note: The figure compares the averages of KL measure by quantile for the benchamrk $D^{[\tau,\tau+10]}_{KL,t}(h)$ and $D^{[\tau,\tau+10]}_{KL,t-j}(h)$ when inflation driver j is removed for j=[Unemployment gap, Oil price, Financial stress index]. Panels B and C show the time series for $D^{[\tau,\tau+10]}_{KL,t}(h)$ and $D^{[\tau,\tau+10]}_{KL,t-j}(h)$ when inflation driver j is removed for j=[Financial stress index]. Gray shaded areas indicate CEPR-dated recessions.

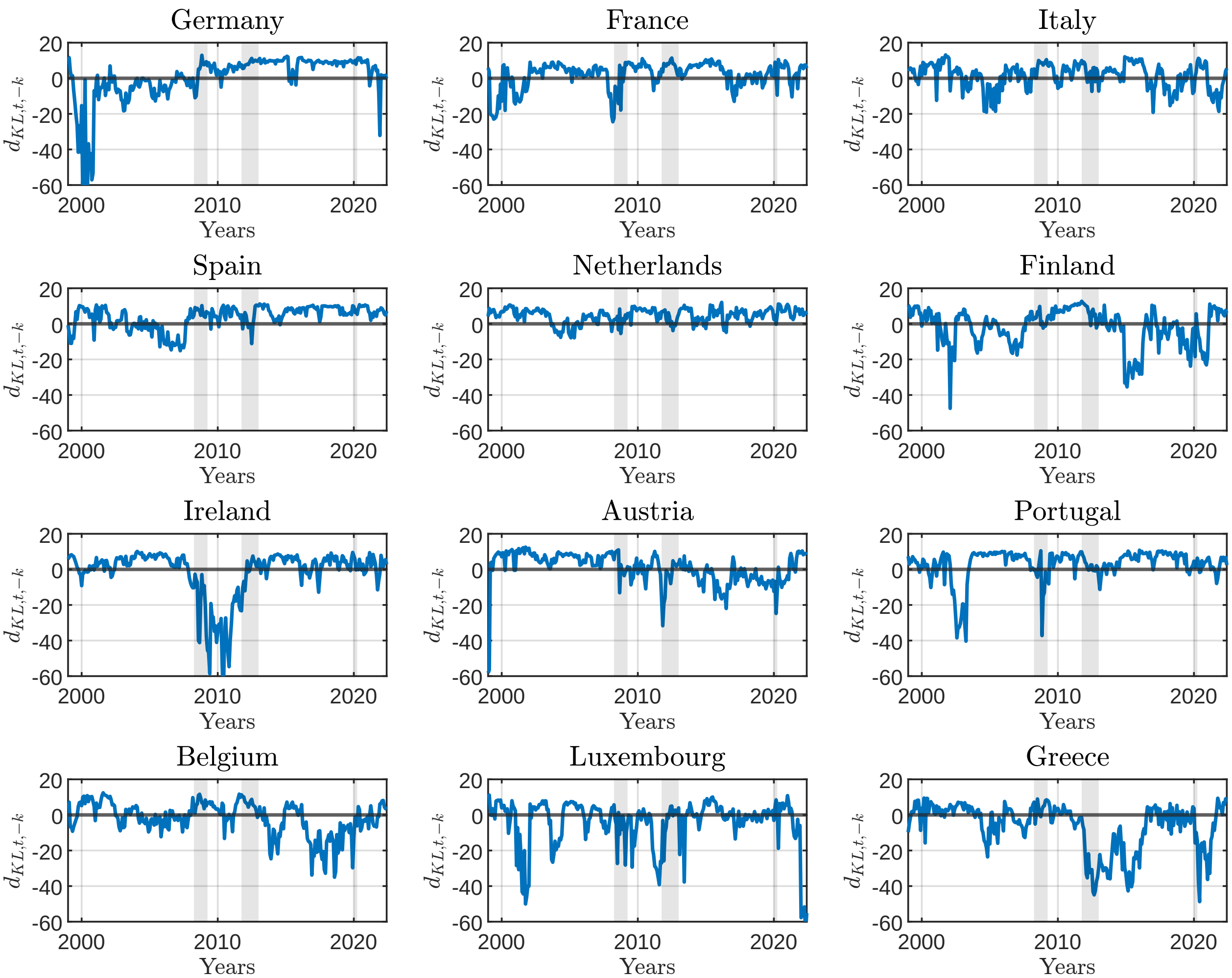
Figure D3. The Risk of Inflation Dispersion by Inflation Driver and Quantile



Note: Panels A to C show the time series for $D^{[\tau,\tau+10]}_{KL,t-j}(h)$ when inflation driver j is removed for j=[Financial stress index]. Gray shaded areas indicate CEPR-dated recessions.

Appendix E. Country contributions to the dispersion of inflation risks: Results for all countries

Figure E4. Country Contributions to the Dispersion of Inflation Risks



Note: The figure shows the values of $d_{KL,t,-k}$, defined by equation (9) for each country k over the sample period. Gray shaded areas indicate CEPR-dated recessions.

Appendix F. List of Variables used for forecasting Exercises

Table F4. List of Variables Included in Z_t to Estimate Macroeconomic and Financial Factors

Variables	*Code Series (ECB SDW)*	*Transf.*
Adjusted loans to euro area private sector	BSI.M.U2.Y.U.A20TA.A.1.U2.2200.Z01.E	2
Monetary aggregate M3	BSI.M.U2.Y.V.M30.X.1.U2.2300.Z01.E	2
Japanese yen/Euro	EXR.M.JPY.EUR.SP00.A	1
Unemployment rate (as a % of labour force)	STS.M.I8.S.UNEH.RTT000.4.000	1
Euribor 3-month	FM.M.U2.EUR.RT.MM.EURIBOR3MD_.HSTA	1
EER-42/Euro	EXR.M.E7.EUR.EN00.A	1
CPI deflated EER-42/Euro	EXR.M.E7.EUR.ERC0.A	1
Dow Jones Euro Stoxx 50 Price Index	FM.M.U2.EUR.DS.EI.DJES50I.HSTA	2
Standard and Poors 500 Composite Index	FM.M.US.USD.DS.EI.S_PCOMP.HSTA	2
HICP - Overall index	ICP.M.U2.Y.000000.3.INX	2
ECB Commodity Price index	STS.M.I8.N.UWIE.CTOTNE.3.000	2
Unemployment rate, Male	STS.M.I8.S.UNEH.RTM000.4.000	1
New passenger car registration	STS.M.I8.Y.CREG.PC0000.3.ABS	2
Industrial new orders; total	STS.M.I8.Y.ORDT.NSC002.3.000	2
Industrial production for the euro area	STS.M.I8.Y.PROD.NS0020.4.000	2
Industrial production; intermediate goods	STS.M.I8.Y.PROD.NS0040.4.000	2
Industrial production; consumer goods	STS.M.I8.Y.PROD.NS0080.4.000	2
Industrial production; energy	STS.M.I8.Y.PROD.NS0090.4.000	2
Industrial production; including construction	STS.M.I8.Y.PROD.NS0010.4.000	2
Industrial production; excl. construction, energy	STS.M.I8.Y.PROD.NS0021.4.000	2
Industrial production; durable consumer goods	STS.M.I8.Y.PROD.NS0060.4.000	2
Industrial turnover, nominal; manufacturing	STS.M.I8.Y.TOVT.2C0000.4.000	2
Retail trade turnover	STS.M.I8.Y.TOVT.NS4701.4.000	2
UK pound sterling/Euro	EXR.D.GBP.EUR.SP00.A	1
EONIA	EON.D.EONIA_TO.RATE	1
U.S. dollar/Euro	EXR.D.USD.EUR.SP00.A	1
HICP; excluding energy and unprocessed food	ICP.M.U2.Y.XEFUN0.3.INX	2
Euribor 1-year	RTD.M.S0.N.C_EUR1Y.E	1
Brent crude oil 1-month Forward	RTD.M.S0.N.P_OILBR.E	2
Consumer Confidence Indicator	RTD.M.S0.S.Y_CSCCI.F	2
Economic Sentiment Indicator	RTD.M.S0.S.Y_ESIND.F	2
U.S. Consumer Price Index	CPIAUCSL (FED FRED)	2
U.S. all Employees, Total Nonfarm	PAYEMS (FED FRED)	2
U.S. 10-Year Treasury Constant Maturity Rate	DGS10 (FED FRED)	1
U.S. Advance Real Retail and Food Services Sales	RRSFS (FED FRED)	2
U.S. 3-Month Treasury Bill	TB3MS (FED FRED)	1
U.S. Unemployment Rate	UNRATE (FED FRED)	1
U.S. ISM Manufacturing PMI	NAPMPMI Index (Bloomberg)	2
1-year inflation forecast	ECB projections	1

Note: Data transformations: 1=first difference; 2=growth rate.