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EUROSYSTÈME

No country is an island. International cooperation and climate change.

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ABSTRACT

In this paper we explore the cross-country implications of climate-related mitigation policies. Specifically, we set up a two-country, two-sector (brown vs green) DSGE model with negative production externalities stemming from carbon-dioxide emissions. We estimate the model using US and euro area data and we characterize welfare-enhancing equilibria under alternative containment policies. Three main policy implications emerge: i) fiscal policy should focus on reducing emissions by levying taxes on polluting production activities; ii) monetary policy should look through environmental objectives while standing ready to support the economy when the costs of the environmental transition materialize; iii) international cooperation is crucial to obtain a Pareto improvement under the proposed policies. We finally find that the objective of reducing emissions by 50%, which is compatible with the Paris agreement's goal of limiting global warming to below 2 degrees Celsius with respect to pre-industrial levels, would not be attainable in absence of international cooperation even with the support of monetary policy.

Keywords: DSGE model, open-economy macroeconomics, optimal policies, climate modelling

JEL classification : F42, E50, E60, F30

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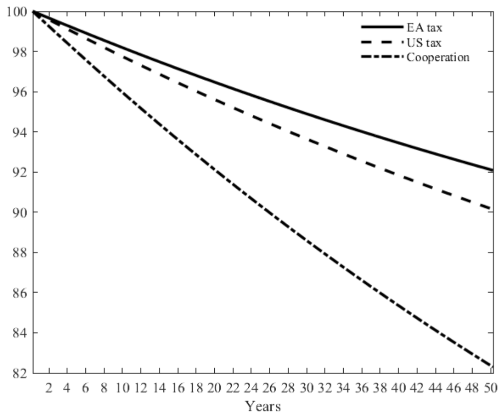
NON-TECHNICAL SUMMARY

The discussion on the impact of climate change and of potential mitigation policies has gained momentum both in academia and policy circles over the last decades. Heatwaves, floods and natural disasters at broad are raising the awareness that the long-neglected costs of adverse climatic events might materialize sooner than expected. Scientific studies, endorsed by international agreements, have estimated that countries should reduce their emissions by about 50% in order to maintain the increase in temperature below 2 degree Celsius over the next century (IMF (2019)). Research networks involving central banks and policy institutions have been created to discuss how to best tackle climate change and how to calibrate mitigation policies. A recent report by the Network for the Greening of the Financial System (NGFS), organized by 87 central banks and supervisory authorities, for example, provides a detailed analysis of the potential impacts of climate change onto the economy and the central banking operations (NGFS (2020)). Despite there exist robust empirical evidences on the macroeconomic costs of higher emission levels (Nordhaus (1994) and Hsiang et al. (2017)), there are relative few structural macro models featuring emission externalities that can be used to analyse the trade-offs of different containment policies. Moreover, most of the existing macro-literature mainly makes use of closed-economy frameworks with no cross-country interaction (e.g., Heutel (2012), Ferrari and Nispi Landi (2020) and Dietrich et al. (2021)). Against this backdrop, our paper provides four new contributions. First, we derive an open economy general equilibrium model where emissions and their spillovers can be studied in a structural framework. In our setting there are two countries, each of which produces "brown" and "green" goods. The two goods are perceived as similar by consumers, but the production of brown goods generates a negative emission externality. In this context, the cross-country dimension is particularly relevant because emissions produced in once country affect also the other. As a consequence, only cooperative actions are successful in reducing climate risk at the global level. In economic terms, this is a coordination problem, as actions produce the maximum benefits only if taken jointly. However, profit-maximizing agents might refrain from doing anything as they would benefit more by waiting for other agents to act. Second, we estimate the model with US and euro area data to study how emissions patterns across the Atlantic have changed over the last 20 years. Notably, in our framework the "social cost" of emissions, which is given by the GDP loss due to emissions in the steady state, amounts to around 1.2% of GDP in the US, a figure which looks more realistic and aligned to the empirical estimates compared to other calibrated models' predictions. Third, we compare different policies that can be deployed to reduce emissions: i) a change in monetary policy objectives, whereby the central bank pursues a double mandate of maximizing welfare and reducing emissions; ii) a change in domestic fiscal policy, whereby fiscal authorities start to directly tax emissions; iii) a change in trade policy, whereby one country implements tariffs targeting polluting imports from the foreign economy. We additionally evaluate the consequences of coordinated and competitive actions by agents. Specifically, we show that the non-cooperative equilibrium is characterized by an insufficient level of taxation on emissions, as both countries attempt to entirely pass the cost of emissions containment on to their respective counterpart. Therefore, only coordinated policies can achieve the climate objective when fiscal and monetary policy interact. Notably, the best policy mix is the one where governments focus on reducing emissions, while the central banks intervene to sterilize the welfare costs of environmental taxation.

Figure: Transition of global emissions stock for different policy configurations



Notes: transition of global emissions stock between the equilibrium without containment policy to the *equilibria* with individual taxation and cooperative taxation. Emission stock is normalised to 100 when the policy is implemented

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Aucun pays n'est une île. La coopération internationale et le changement climatique

RÉSUMÉ

Dans ce document de travail, nous explorons les implications transnationales des politiques d'atténuation liées au climat. Plus précisément, nous construisons un modèle DSGE à deux pays et deux secteurs (brun et vert) avec des externalités de production négatives découlant des émissions de dioxyde de carbone. Nous estimons le modèle en utilisant des données des États-Unis et de la zone euro et nous caractérisons les équilibres d'amélioration du bien-être dans le cadre de politiques de limitation alternatives. Trois implications politiques principales émergent : i) la politique fiscale devrait se concentrer sur la réduction des émissions en prélevant des taxes sur les activités de production polluantes; ii) la politique monétaire ne devrait pas se concentrer sur les objectifs environnementaux, mais plutôt se tenir prête à soutenir l'économie lorsque les coûts de la transition environnementale se matérialisent ; iii) la coopération internationale est cruciale pour obtenir une amélioration de Pareto dans le cadre des politiques proposées. Enfin, nous constatons que l'objectif de réduction des émissions de 50 %, qui est compatible avec l'objectif de l'accord de Paris de limiter le réchauffement climatique à moins de 2 degrés Celsius par rapport aux niveaux préindustriels, ne serait pas atteignable en absence de coopération internationale, même avec le soutien de la politique monétaire.

Mots-clés : modèle DSGE, macroéconomie en économie ouverte, politiques optimales, modélisation du climat

1 Introduction

The discussion on the impact of climate change and of potential mitigation policies has gained momentum both in academia and policy circles over the last decades. Heatwaves, floods and natural disasters are raising the awareness that the long-neglected costs of adverse climatic events might materialize sooner than expected. Scientific studies, endorsed by international agreements, have estimated that countries should reduce their emissions by about 50% in order to maintain the increase in temperature below 2 degree Celsius over the next century (IMF (2019)). Networks among central banks and policy institutions are now discussing how to best tackle climate change and how to calibrate mitigation policies. NGFS (2020), for example, provides a detailed analysis of the potential impacts of climate change onto the economy.

In spite of the wide and growing empirical literature that tries to quantify the costs of climate events and to assess the relationship across emissions, climate disasters and economic performance, few structural models have been developed in this field[1]. Such frameworks could be anyways useful to study the implications of policies that have arguably never been implemented before and, therefore, cannot be ascertained on the basis of past data. Among the existing contributions on this topic, Heutel (2012) is the first to provide a structural model where emissions are endogenous and affect output.[2] One advantage of using structural models is that they can be used to formally study optimal policy problems and identify trade-offs between different specifications of the policies of interest. The literature on optimal policy in DSGE model is vast, including the pioneering work of Woodford (2003) on monetary policy, but only recently these tools have been applied to the design of climate change mitigation policies (Benmir et al. (2020), Kotlikoff et al. (2020)). Against this backdrop, our paper provides five relevant contributions. First, we show that there are relevant non-linearities in the relationship between carbon emissions, a common measure of climate externalities, and macro variables. For example, when the level of emissions is low there is a positive correlation between CO_2 and GDP, because more output leads to more emissions, but the emission stock is too low to generate sizeable climate events. When the stock of CO_2 is high, on the contrary, that correlation drops because more emissions directly increase the severity of climate shocks leading to GDP losses. These results

[1] Notably, there are several empirical contributions that focus on the assessment of physical and transitional climate risks in the financial sector. See, for instance, Pagliari (2021) for a study of the relationship between climatic adverse events and banking performance in the euro area, or Allen et al. (2020) for the setup of an analytical framework to quantify the impacts of climate policy and transition narratives on economic and financial variables that are necessary for financial risk assessment. Finally, refer to Tol (2009) for a thorough survey of the early theoretical macroeconomics literature on climate change.

[2] There indeed exists a wide literature on the quantification of the cost of climatic events, for example Nordhaus (2008). However models in the spirit of DICE or FUND have been developed to account for several sources of pollution and types of climatic events, but often rely on reduced-form equations for production and consumption side of the economy, which make them less appealing than structural models for the welfare analysis of policies.

suggest that empirical models, if anything, underestimate the real costs of climate externalities on full sample estimations. Second, we derive an open economy general equilibrium model where emissions and their spillovers can be studied in a structural framework. Specifically, we set up a rich two-country two-sector model, where, on the one hand, "brown" production generates a negative environmental externality that is detrimental to both domestic and foreign output, while, on the other hand, "green" production does not. This cross-country dimension is crucial in the debate on climate change, because only a fall in the *global* stock of emission would reduce the likelihood of a "climatic disaster", whereas isolated actions might result insufficient. In economic terms, this is a coordination problem, as efforts produce the maximum benefit only if taken jointly. However, profit-maximizing agents might refrain from doing anything as they would benefit more by waiting for other agents to act, without bearing any direct cost.

Third, we estimate the model with US and euro area data to study how emissions patterns across the Atlantic have changed over the last 20 years. In our framework the "social cost" of emissions, which is given by the GDP loss due to emissions in the steady state, amounts to around 1.2% of GDP in the US, a figure which looks more realistic and aligned to the empirical estimates (Hsiang et al. (2017)) compared to other models' predictions (Heutel (2012)). Fourth, we evaluate the implementation of three alternative policies to reduce emissions: i) a change in monetary policy objectives, whereby the central bank pursues a double mandate of maximizing welfare and reducing emissions; ii) a change in domestic fiscal policy, whereby fiscal authorities start to directly tax emissions; iii) a change in trade policy, whereby one country implements tariffs targeting polluting imports from the foreign economy. We show that both monetary policy and tariffs are not effective in reducing emissions, whereas domestic taxation can achieve the objective. Against this background, we determine the Nash equilibrium of a game where each country optimally sets the domestic environmental tax by responding to the opponent's choice. We show that such equilibrium is characterized by an insufficient level of taxation on emissions, as both countries attempt to entirely shift the cost of emissions containment to their respective counterpart. Therefore, only coordinated cross-country policies can attain the appropriate reduction in global emissions.

We then characterize the *incentive compatible* policy mix, defined as the policy which reaches the climate objective without reducing welfare in both countries, and find that it is characterized by a combination of fiscal and monetary policies. Notably, under such policy governments should focus on reducing emissions while the central bank should intervene to reduce the welfare costs of environmental policies. This joint intervention only can ensure that the climate policy is also welfare compatible. The reason is twofold: i) the pattern of emissions is impacted more strongly by altering the incentives of firms and inducing them to shift to a greener production. Fiscal policy has therefore an advantage in tackling climate change compared to monetary policy, which in turn reduces fluctuations around the growth path of the economy; ii) monetary policy plays a crucial role in stabilizing the economy once climate policies are implemented, thus minimizing their social costs. When the environmental tax is introduced, indeed, the relative volatility of inflation and output changes thus providing the monetary authority with the possibility to adjust its optimal reaction function, accommodate the green transition and improve welfare.

Finally, we show that the equilibrium under international cooperation is superior to the non-cooperative Nash equilibrium and that countries cannot credibly threaten to retaliate if their partner deviates from environmental agreements. Our framework could then prove useful also to address some policy issues that have become relevant in the current international debate (NGFS (2020), McKibbin et al. (2020), Dees and Weber (2020)).

1.1 Related literature

There exists a growing literature incorporating the effects of climate change, as well as the potential economic consequences of containment policies, into workhorse macroeconomic models. These models, like ours, depart from standard DICE frameworks, such as the one of Nordhaus (2017)[3]. DICE-type models have been developed to determine the social cost of carbon (SCC), i.e. the economic cost caused by an additional ton of carbon dioxide emissions or its equivalent, by accounting for several sources of pollution at the same time and different dimensions of climate change such as air and sea temperatures. To preserve tractability, however, the framework needs to rest on some simplifying assumptions: there are no assets except for investments, which are given by the difference between output and consumption; there are no wages; prices and interest rates are constant; the world economy is modelled as a unique block and many macroeconomic relations are included via reduced-form equations[4]. These characteristics make the DICE setting less appealing for optimal policy analysis as it abstracts from agents' preferences and expectations, trade-offs across assets and proper inter-temporal investment-consumption decisions. Moreover, DICE models are generally simulated starting from some given initial conditions. In other terms, they do not have a properly defined steady state which, in most monetary and fiscal models, is the starting point for the evaluation of policies. Finally, structural models can be estimated with standard methods whereas DICE models are typically calibrated.

[3]See also: The DICE-RICE model by William Nordhaus.

[4]Another popular model for the consumption of environmental resources, the FUND model, is also based on reduced-form equations (Waldhoff et al. (2014)).

For all these reasons a new generation of macro-models accounting for climate change has been developed in recent years. Our paper fits within this growing research stream, by extending it to a multi-country environment.

Some of the existing contributions aim at assessing the impact of fiscal policies that can be deployed to address climate change. Heutel (2012), for instance, is the first one to include emission externalities in a DSGE model to evaluate optimal containment policies along the business cycle. The main finding is that both quota and tax policies should be pro-cyclical, with the tax rate and the emissions quota decreasing during recessions. Similarly, Benmir et al. (2020) set up a theoretical model to study the optimal design of a carbon tax when environmental factors, such as CO_2 emissions, directly affect agents' marginal utility of consumption. Using asset pricing theory, they show that the optimal taxation policy is pro-cyclical: the carbon tax should be increased during booms to "cool down" the economy and should be decreased to stimulate it in recessions. Our paper differs from Benmir et al. (2020) in two respects: from a methodological standpoint, we do not directly include emissions in the households' utility function; context-wise, we aim at assessing the *international* effects of environmental policies. Annicchiarico and Di Dio (2015), instead, develops a New Keynesian model to study the economy under different environmental policy regimes highlighting that the optimal environmental policy response to shocks depends on price stickiness and on the monetary policy conduct.

Other contributions focus more on the potential role of monetary policy, given that climate change and its mitigation can have substantial repercussions on the conduct of monetary policy along several dimensions[5]. Dietrich et al. (2021), for instance, explore the so-called *expectation channel* of climate change, whereby agents' expectations about future climatic disasters can negatively impact the economy today via a drop in the natural rate of interest, if the central bank is unwilling or unable to adjust its policy in a timely manner. Given the limitations that conventional monetary policy might face, Ferrari and Nispi Landi (2020) build a DSGE model where the central bank can *temporarily* modify its balance sheet to buy bonds issued by non-polluting firms. This Green QE is found to have limited effects in reducing the stock of emissions and to produce positive, yet small, welfare gains.

Another aspect of the debate concerns the estimation of the real economic costs of emissions. Tol (2009) provides an overview of the different estimates of the earliest literature, with losses in GDP ranging from -4.8% (Nordhaus (1994)) to -0.1% (Maddison (2003)). More recently, Burke et al. (2015) have found that climate change is expected to reshape the global economy by

[5] The NGFS (2020) report outlines four crucial aspects to be considered in this regard: i) the effect on the key macroeconomic variables that are targeted by monetary policy; ii) the effect on the main transmission channels as well as on the assessment of the policy space; iii) the expansion of the central banks' analytical toolkits to include climate-related risks; iv) the diversified impact of climate change on the different monetary regimes.

reducing global economic output and possibly amplifying existing global economic inequalities, with an estimated impact on global income of -23% by 2100. Similarly, Hsiang et al. (2017) underscore the role of climate change in exacerbating income inequalities and estimate that, by the late 21st century, poorest countries are projected to experience damages between 2% and 20% of country income under the current emissions scenario. Nordhaus (2017), by revising the estimates provided by Nordhaus (2008), quantifies the cost of CO_2 emissions in the US as being equal to 32 dollars per metric cube (3.2% of GDP). As to the effects on emerging markets, Bombardini and Li (2020) find that a one standard deviation increase in the pollution content of exports in China raises infant mortality by 4.1 deaths per thousand live births, which is about 23% of the standard deviation of infant mortality change. Finally Desmet et al. (2021) use a spatial model to quantify the effects of rising sea levels that are estimated to range 0.16-0.25 p.p. of GDP in present values. In this respect, our model produces estimates of the cost of climate externalities (in GDP terms) that are aligned to the existing empirical measures.

Another strand of research focuses on the potential consequences of policies enacted to contain the negative effects of climate change. Brock et al. (2013), for instance, construct a computable general equilibrium model where the degree of spatial differentiation of optimal taxes depends on heat transportation. Keen and Kotsogiannis (2014) analyse the possibility for countries to implement some form of border tax adjustments (BTAs) to countervail distortions stemming from different carbon pricing and show that BTAs' efficiency depends on whether climate policies are pursued by carbon taxation or by cap-and-trade. In a similar vein, Larch and Wannera (2017) show that the introduction of carbon tariffs in a multi-sector, two-factor gravity model reduces welfare in most countries, with the effect being more pronounced in poorer economies. However, they also show that carbon emissions can be shifted from these to richer countries. Nordhaus (2015) extends the DICE framework to more countries and shows the conditions under which a "climate club" can be established in that framework.[6] In a more recent contribution, Hambel et al. (2021) construct a DSGE model featuring a social cost of carbon, which measures the externalities incurred into by emitting one ton of carbon dioxide to the atmosphere. Accounting also for the feedback effects of SCC on temperature dynamics, they show how the optimal abatement strategy and, hence, SCC crucially depend on whether temperature has a negative impact on either the level or the growth rate of output. Our paper also fits into this particular literature, as it analyses the open-economy implications of containment policies. Specifically, we focus on the risks stemming from a lack of international cooperation, thus highlighting the

[6] The literature on climate policies is clearly larger, and cannot be summarized here. Other important contributions, discussing different domestic and cross-country policies, are Larch and Wanner (2017), Böhringer et al. (2016), Aichele and Felbermayr (2012), Eichner and Pethig (2011), Elliott et al. (2010).

importance of cross-country coordination when it comes to pursue a commonly agreed environmental objective.

The remainder of the paper is structured as follows: Section 2 provides some empirical evidence on the relationship between economic performance and emissions; Section 3 describes the main features of the structural model; Section 4 reports the estimation strategy and the model's posterior results; Section 5 discusses the different containment policies; Section 6 concludes.

2 Empirical evidence

In this section we provide some empirical evidence which is instrumental for supporting the underlying assumptions of our theoretical framework. In particular, Section 2.1 investigates the empirical regularities characterizing the link between CO_2 emissions and economic performance in the euro area and the US, which are the economies of reference in our model. The same section also presents some anecdotal evidence on the possible discrepancy in the returns to capital across firms operating in brown and green sectors.

In Section 2.2 all these stylized facts are assessed in a more structural context, by setting up two non-linear Vector Autoregression (VAR) models, one for each country, whose parameters are allowed to switch across two different Regimes depending upon past growth rate in annual CO_2 emissions. The results of the exercise provide a rationale for some of the assumptions we make in Section 3.

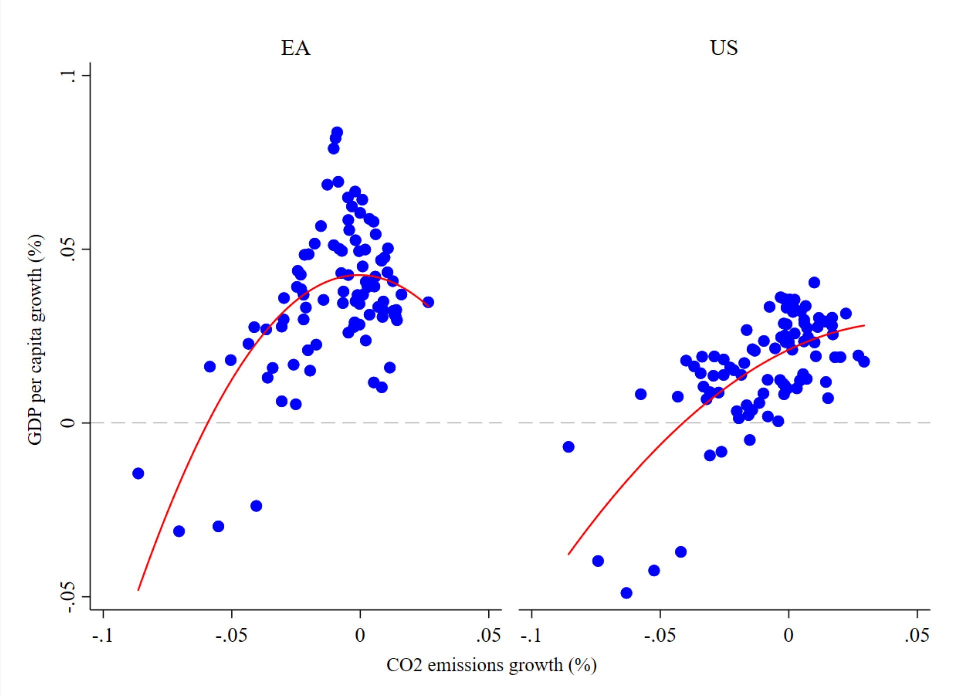
2.1 Some stylized facts

A crucial element in our theoretical model, which is also considered as a discriminatory factor across different economies, is the impact of CO_2 emissions on aggregate production. An empirical analysis of the relationship between production and CO_2 emissions could provide us with useful insights about the assumptions to be made about the functional form of such link.

Figure 1 depicts the correlation between the GDP per capita y-o-y growth and the annual growth in CO_2 emissions for the euro area and US over the period 1995Q1-2018Q4. A first look at the data seems to suggest that in both economies the relationship between the two variables is best described via a quadratic (inverse) function, though this is more the case for the euro area compared to the US. This phenomenon, also referred to as the "environmental Kuznets curve" (EKC), has been already discussed by the literature, but conclusions are mixed[7]. Dées

[7] The EKC is named after Kuznets (1955), who hypothesized that income inequality first rises and then falls as economic development proceeds. Alternative hypotheses concerning the relationship between economic growth and emissions are: 1. *the Brutland curve hypothesis*, whereby the relationship is best described by a proper U-shaped function; 2. *the environmental Daly curve hypothesis*, whereby there are no turning points in

Figure 1: Relationship between GDP per capita growth and CO_2 emissions growth in the euro area and US.



Notes: CO_2 emissions per capita are computed as the ratio between CO_2 emissions in thousands of tonnes and the total population in thousands.
Sources: OECD, authors' computations

(2020), for instance, provides evidence of income-related threshold effects in the relationship between emissions and growth for a panel of 142 countries[8]. In a more recent contribution, Tol (2021) sets up a stochastic frontier model with climate in the production frontier and weather shocks as a source of inefficiency for a sample of 160 countries.

On the other hand, Grubb et al. (2004) show that the statistical analysis which the EKC hypothesis rests upon is not robust, thus discarding the assumption of a common inverted U-shaped pathway that countries follow as their income rises.

We delve deeper into this particular aspect by setting up an exponential model of the form $y_t = \alpha x_t^{\beta}$, where y_t is GDP per capita and x_t is the amount of CO_2 emissions per capita. By applying the log operator on both sides, the equation becomes:

$$\tilde{y}_t = \tilde{\alpha} + \beta \tilde{x}_t \tag{2.1}$$

where $\tilde{\alpha} \equiv \log\alpha$, $\tilde{y}_t^i \equiv \log y_t$, $\tilde{x}_t \equiv \log x_t$. We can get a precise estimation of the curvature of the quadratic function, β, by running an OLS regression. Results, reported in Table 1, confirm that the parameter describing the curvature is significantly bigger for the euro area compared

the relationship between production and emissions.

[8]In a slightly different empirical setting, Wang (2013) shows similar results in a panel of 138 countries.

to the US. In particular, estimates seem to indicate that an unitary increase in emissions entails a twofold loss in production in the euro area, while the same loss is less than proportional in the US, as also confirmed by the F tests performed on the β's.

Table 1 also reports regression results using yearly growth rates[9], which are in line with what shown in Figure 1. Specifically, while the slope coefficients are positive for both the euro area (0.527) and the US (0.510), we also find evidence of a reversal in the sign of coefficients, with estimates dropping to -0.63 and -0.26 for the euro area and the US respectively when the annual emission growth rates are above 0[10].

Table 1: OLS estimates for Equation (2.1)

Variables \ Country	EA (1)	EA (2)	US (1)	US (2)
Emissions	-2.117***	0.527***	-0.775***	0.510***
	(0.114)	(0.106)	(0.0543)	(0.0915)
F tests	$H_0 : \|\beta\| = 2$	$H_0 : \|\beta\| = 1$	$H_0 : \|\beta\| = 1$	$H_0 : \|\beta\| = 1$
P-value	0.30	0.00	0.00	0.00
Observations	96	92	96	92
R^2	0.676	0.258	0.565	0.443

Notes: (1) Equation in levels; (2) Equation in growth rates. Robust standard errors in parentheses. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$.

Another aspect to be considered concerns the differences between the "brown" and "green" production processes, especially as far as profitability is concerned. As explained in Section 3 below, our model postulates that the two sectors feature different returns to capital. Once again, a preliminary analysis of existing data might provide us with a guidance about the extent and direction of such difference. Notably, we look at the performance of green and brown companies in the US and in the euro area (Figure 2). In the US, greener companies constantly feature a lower Return on Assets (ROA) compared to brown companies, whereas the same metric seems more aligned across the two groupings in the euro area, with green companies outperforming brown ones as of the mid-2000s, especially in periods of economic expansion (2005-2008) or recovery (2011-2016).

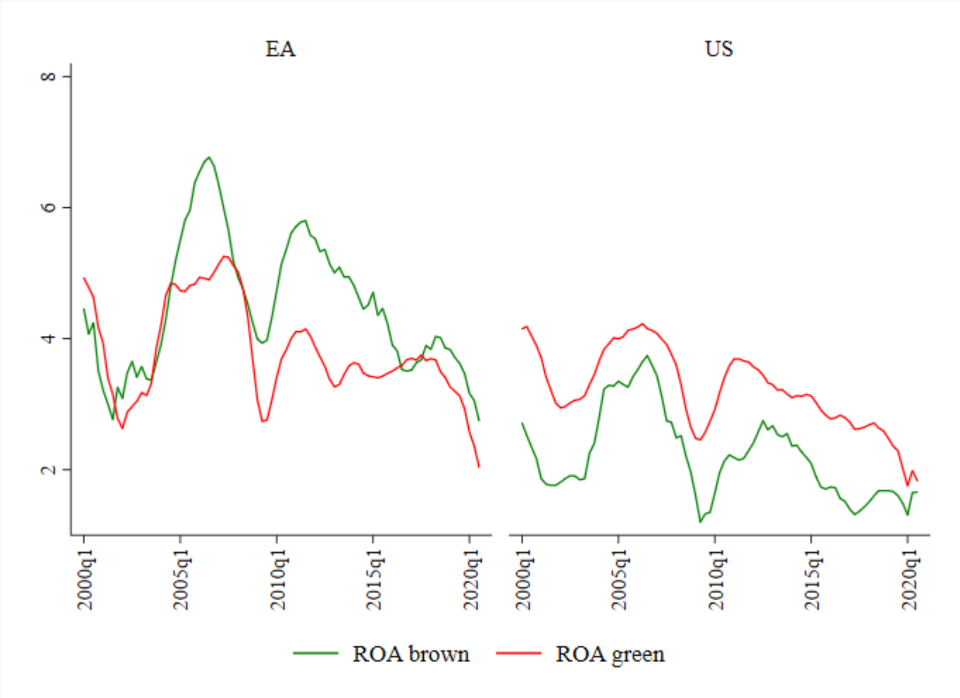
2.2 A more structural approach

Evidences provided in Section 2.1 above might be the result of spurious correlations, rather than of a proper structural link across emissions, output and the profitability of green and brown firms. In particular, OLS estimates of Equation (2.1) might be biased by reverse causality

[9] Equation (2.1) can be easily expanded to annual growth rates. Notably, taking the 4-th order difference on both sides leads to: $\hat{y}_t = \beta \hat{x}_t$, with $\hat{z}_t \equiv \tilde{z}_t - \tilde{z}_{t-4}$, $z \in \{y, x\}$.

[10] However, the estimates are not significantly different across the two economies.

Figure 2: Median ROA at green and brown companies.



Notes: Annual moving averages. Return on Assets green (brown) is computed as the ratio between profit/losses after tax and assets at listed companies with highest (lowest) GHG Scope1 and Scope2 values, expressed as share of revenues, according to a quartile classification.
Sources: Bloomberg, authors' computations

between emissions and GDP. In this section, we make use of a structural approach to produce more grounded empirical evidence on the relationship between CO_2 emissions and economic performance, which is one of the main elements in the model discussed in Section 3.

The relationship between emissions and GDP might be highly non-linear. For a low stock of CO_2, the correlation between emissions and GDP might be positive: higher production increases emissions, but the level of pollution in the atmosphere might not be high enough to trigger negative climate events thus reducing output. Only when CO_2 levels are high enough more emissions might trigger severe climate events hence reducing GDP. This non-linear relationship is indeed suggested by both Figure 2 and estimation of Equation (2.1). To structurally test this hipothesis we set up two separate *threshold* Vector Autoregression (TVAR) models with the following reduced form:

$$\begin{aligned} \boldsymbol{Y}_{i,t} &= S_t[\boldsymbol{B}_0^1 + \boldsymbol{B}_1^1 \mathbf{Y}_{i,t-1} + \cdots + \boldsymbol{B}_p^1 \mathbf{Y}_{i,t-p} + \boldsymbol{u}_{i,t}] \\ &\quad + (1 - S_t)[\boldsymbol{B}_0^2 + \boldsymbol{B}_1^2 \mathbf{Y}_{i,t-1} + \cdots + \boldsymbol{B}_p^2 \mathbf{Y}_{i,t-p} + \boldsymbol{u}_{i,t}], \end{aligned} \tag{2.2a}$$

where

$$S_t = \begin{cases} 1 \iff z_{t-d} \leq z^* & \text{(Regime 1)} \\ 0 \text{ otherwise} & \text{(Regime 2)} \end{cases} \tag{2.2b}$$

In Equation (2.2), $i \in \{US, EA\}$ and $\boldsymbol{Y}_t$ includes: e_t, the log-level of total emissions; y_t, the annual real GDP growth; π_t, the annual CPI inflation; roa_t^g and roa_t^b, the Return on Assets in the green and brown sectors respectively; r_t, the Wu & Xia's shadow interest rate[11]. The model allows the system to shift between two distinct regimes, depending upon the level of the variable z_{t-d} with respect to an unknown threshold level, z^*. In our setting, z_{t-d} is the (lagged) annual emissions growth rate, while d and z^* are parameters to be estimated. Moreover, we assume that $\boldsymbol{u}_{i,t} \overset{\text{i.i.d.}}{\sim} \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma})$, with $\boldsymbol{\Sigma} = \text{diag}(\sigma_1^2, \sigma_2^2, \ldots, \sigma_N^2)$. The identification of structural shocks in both regimes is achieved recursively via the Choleski decomposition of the reduced-form variance-covariance matrix, $\boldsymbol{\Sigma}$[12].

The aim of this empirical exercise is to characterize the behaviour of the economy in both the euro area and the US, conditional on the past growth rate of CO_2 emissions. Following Alessandri and Mumtaz (2017), we estimate Equation (2.2) by using natural conjugate priors for the VAR parameters in both regimes, as proposed by Litterman (1986), Sims and Zha (1998) and Bańbura et al. (2010)[13]. Notably, we choose identical priors for the two regimes. In addition, we set $p = 4$ and we base our inference on a total of 100000 draws from the posterior, with a burn-in of 50000 draws.

Given the recursive identification strategy, the ordering of the endogenous variables crucially influences the final results. Therefore, we get a more precise indication as to which elements to put first in $\boldsymbol{Y}_t$ by running pairwise Granger-causality tests across the endogenous variables. Results, summarized in Table 2, suggest the following ordering: $\boldsymbol{Y}_t = [e_t, y_t, \pi_t, roa_t^b, roa_t^g, i_t]$.

Table 2: Granger causality tests

Variables *EA*/*US*	π_t	e_t	y_t	roa_t^b	roa_t^g
e_t	← / ↔				
y_t	↔ / ←	→ / →			
roa_t^b	← / ←	- / -	↔ / ↔		
roa_t^g	← / →	- / -	← / -	- / -	
r_t	- / ↔	← / -	↔ / -	↔ / →	- / -

Notes: → (←): the row (column) variable Granger-causes the column (row) variable; ↔: Granger causality runs in both ways; -: no meaningful Granger causality is found in either direction between two variables.

[11] Time series are demeaned and standardized. See also Appendix A for an overview of data sources.

[12] This approach works under the assumption that the variance-covariance matrix of the structural shocks is given by the identity matrix.

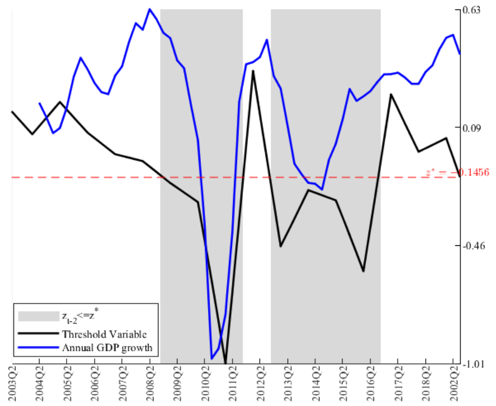
[13] See Alessandri and Mumtaz (2017) for the technical details on the estimation methodology.

2.2.1 Results

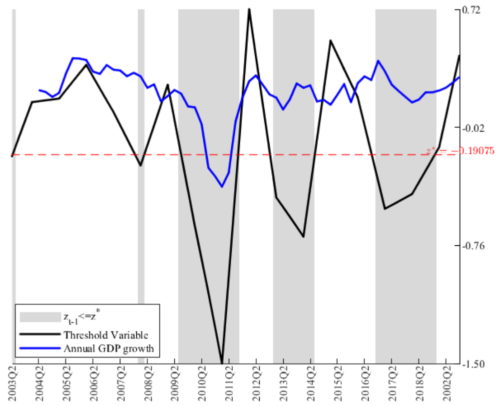
The estimation algorithm delivers a lag on the threshold variable, d, of 2 both for the US and the euro area. The threshold level for annual emissions growth, z^*, is estimated to be -0.14% for the euro area and -0.19% for the US. Figure 3 below depicts the identification of the two different regimes, with low and high level of emissions growth, in the two economies over time. Regime 1, which is associated with lower growth in emissions, is broadly coinciding with periods of economic slowdowns in both cases.

Figure 3: Regime identification

(a) Euro Area



(b) US

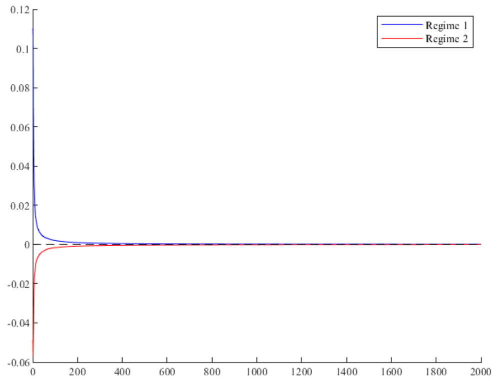


Notes: Grey shaded areas indicate periods where emissions growth has been below the estimated threshold, identified as Regime 1.

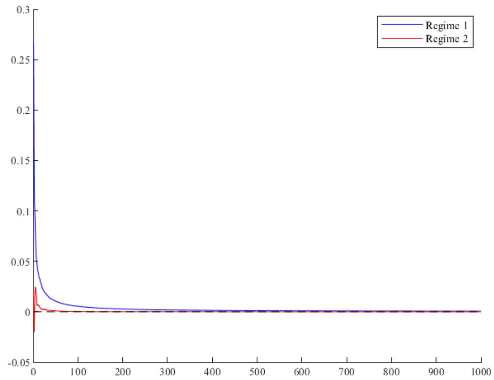
A more interesting set of results is provided by the posterior estimates of the contemporaneous coefficients, i.e. how strong is the *contemporaneous* relationship between endogenous variables, that are recursively identified in the two regimes. Figure 4 below plots the posterior estimates of the emissions coefficients in the GDP equation, for both the euro area (Figure 4a) and the US (Figure 4b). There is a significant difference in the coefficient estimates across the two regimes in both countries (see Table 3). Notably, in Regime 1 (lower past annual emissions growth) the sign remains positive after the convergence, thus implying that an increase in emissions has a positive effect on output. In Regime 2 (higher past annual emissions growth), on the other hand, the same effect turns not significant in the US and even negative in the euro area, which seems to suggest that there can be negative spillovers of increasing emissions on a country's economic performance. Results therefore confirm the preliminary evidences produced by less structural approaches in Section 2.1 and can be used to inform the functional assumptions in the theoretical model (see Section 3). Moreover, results for inflation suggest that more emissions have limited effects on prices under both regimes. This stylized fact is also captured by the impulse response of the structural model presented later.

Figure 4: Convergence check for posterior estimates of contemporaneous coefficient of CO_2 emissions in GDP equation - $\alpha_{1,2}$

(a) Euro Area



(b) US



Notes: Solid lines depict the rolling means of posterior estimates over 2000 draws that are randomly selected among the posterior draws. Values on the x-axis indicate the number of draws.

Table 3: Posterior estimates of contemporaneous coefficients of CO_2 emissions - $\alpha_{1,j}$, $j = 1, \ldots, 6$

Coefficients	$\alpha_{1,e}$	$\alpha_{1,y}$	$\alpha_{1,\pi}$	α_{1,roa^b}	α_{1,roa^g}	$\alpha_{1,i}$
Euro Area						
Regime 1	**0.077***	**0.091***	0.002	-0.071	0.007	**0.057***
Regime 2	**0.043***	**-0.083***	**0.009***	**-0.086***	**-0.084***	**0.027***
Difference	**-0.034***	**-0.175***	0.008	-0.014	-0.095	-0.03
United States						
Regime 1	**0.034***	**0.274***	-0.006	**-0.197***	-0.043	**0.027***
Regime 2	**0.041***	0.009	0.007	**0.169***	**-0.119***	-0.008
Difference	**0.007***	**-0.271***	0.014	**0.371***	-0.072	**-0.036***

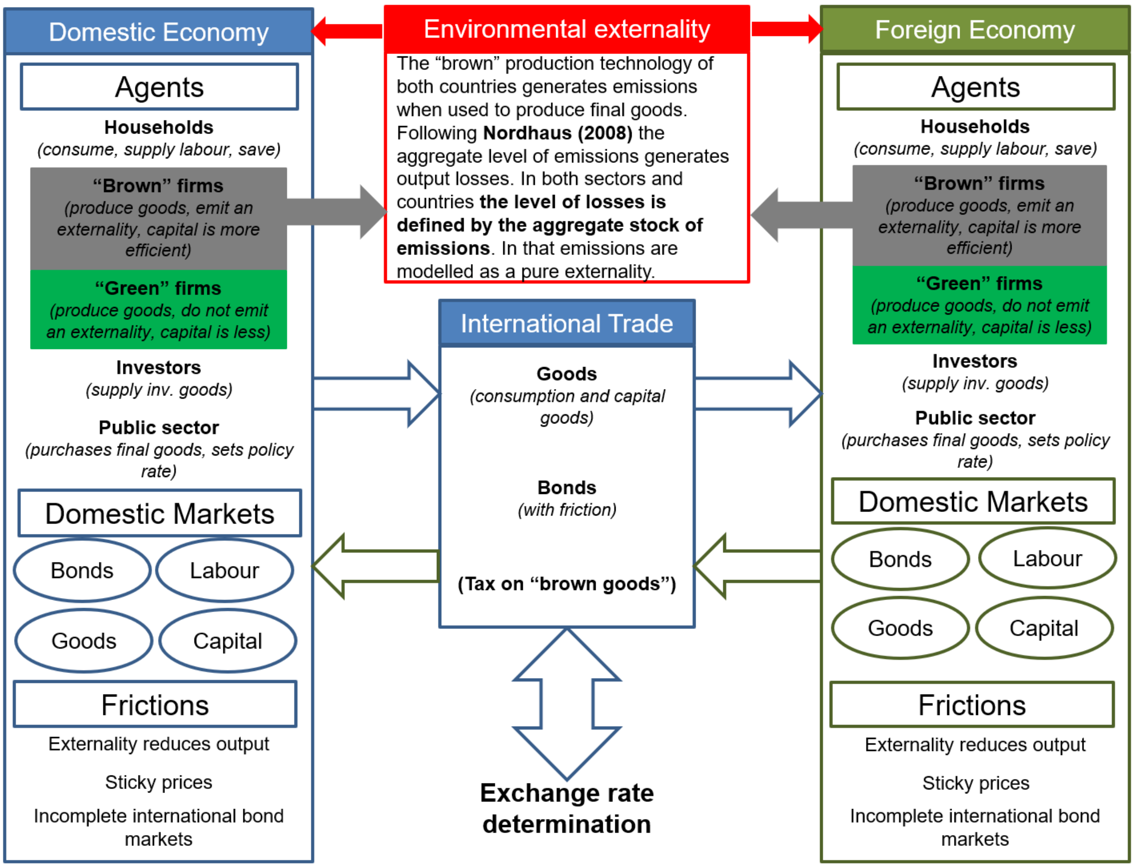
Notes: *significant at the 68% confidence level based on HPD sets. The difference is computed as $\alpha^2_{1,j} - \alpha^1_{1,j}$ $\forall j = 1, \ldots, 6$, with the superscript indicating the regime.

3 The model

Our baseline two-country model (home vs foreign) includes households, a financial sector, competitive producers, retailers, the government and the central bank (Figure 5).

In each country there is a continuum of firms producing with brown and green technology, indexed by k and j respectively. Brown and green goods, which are bundled together in final consumption, both generate the same marginal utilities to households, i.e. they are identical to consumers, but are produced with different technologies. Based on the evidence provided in Section 2, indeed, we assume that the aggregate production of brown goods across countries generates negative externalities, i.e. the CO_2 emissions, which are detrimental to the whole production. As k firms are atomistic in nature, they take the aggregate level of brown production as exogenous. The inclusion of emissions allows to characterize the economic costs of climatic

Figure 5: Model setup



events in the spirit of Nordhaus (2008) and Stern (2008). The estimation of the model, which uses data on sector-specific productivity and global emissions, would allow to quantify those costs in terms of output.

The remainder of the model includes several standard features of international macro models in the spirit of Eichenbaum et al. (2021). Notably, households consume, supply labor and save by purchasing domestic and foreign bonds. International capital markets are incomplete, which entails that: i) households incur in frictions when buying foreign bonds; ii) and the UIP condition does not hold. Intermediate goods are bundled together by retailers, who sell them on the final domestic and foreign markets with some degree of monopoly power. Prices are assumed to be sticky, with retailers that are able to optimally reset prices in each period with probability ξ.

The central bank follows a Taylor-type rule and the government chooses the level of public spending. When discussing climate policies we allow the government to intervene to steer the economy towards a "greener" production by imposing taxes on domestic and imported brown goods. Finally, the exchange rate is determined by interest rate differentials, international trade, prices and taxes.

3.1 Main sectors

In what follows, we present the key equations characterizing the domestic economy only, as the foreign economy is perfectly symmetric. Specifically, in this section we introduce the main building blocks of the model, i.e. consumers' preferences and production, while leaving the complete model with all the derivations in Appendix B.

3.1.1 Consumers

The non-production block of the economy is modelled as in Eichenbaum et al. (2021). In each period t, households' utility depends on aggregate consumption and labor supplied:

$$U_t = E_t \sum_{j=0}^{\infty} \beta^j \left[e_t^c \ln\left(C_{t+j} - hC_{t+j-1}\right) - \frac{\chi}{1+\phi} L_{t+j}^{1+\phi} \right]. \tag{3.1}$$

We assume that there are habits in consumption to generate reasonable inter-temporal discount factors as in Campbell and Cochrane (1999, 2000). Households can also invest in domestic capital as well as domestic and foreign bonds. Their budget constraint is:

$$\begin{aligned} &P_t C_t + B_t^H + NER_t B_t^F + \sum_{i=b,g} P_t I_t^i \\ &\leq \sum_{i=b,g} W_t^i L_t^i + R_t B_{t-1}^H + R_t^* NER_t B_{t-1}^F - \frac{\phi^B}{2} \left(\frac{NER_t B_t^F}{P_t} \right)^2 P_t + \sum_{i=b,g} P_t R_t^{k,i} K_{t-1}^i + \Pi_t \end{aligned} \tag{3.2}$$

where $i \in \{b, g\}$ denotes the green and brown sectors respectively. Furthermore, C is aggregate consumption, L represents total labor supply, B^H and B^F stand for the overall amount of domestic and foreign bonds, NER is the nominal exchange rate, W are nominal wages, P is the aggregate price level and Π are profits net of lump-sum taxes. Households can invest in brown (b), or green (g) companies, with I^*, K^* and $R^{k,*}$ denoting investments, capital and capital returns on the two assets respectively. International bond markets are not complete, which implies that the UIP conditions does not hold. As in Christiano et al. (2005), we assume that there are convex investment adjustment costs[14].

Final goods are bundled together by retailers which buy them from perfectly competitive producers and sell them with some degree of monopoly power on the final good market. The final

[14] The full optimization problem and first order conditions are reported in Appendix B.1.

good aggregator for consumption is:[15]

$$C_t = [\omega\Upsilon]^{1-\varrho}\left(C_{H,t}^{b}\right)^{\varrho} + [\omega(1-\Upsilon)]^{1-\varrho}\left(C_{H,t}^{g}\right)^{\varrho} + \\ + [(1-\omega)\Upsilon]^{1-\varrho}\left(C_{F,t}^{b}\right)^{\varrho} + [(1-\omega)(1-\Upsilon)]^{1-\varrho}\left(C_{F,t}^{g}\right)^{\varrho}. \tag{3.3}$$

C_i^b and C_i^g, $i \in \{H, F\}$, are consumption goods produced with the brown and green technology respectively. ω is the home bias, Υ is the share of brown goods in consumption[16] and ϱ the elasticity of substitution between domestic and imported goods. According to Equation (3.3) brown and green goods give the same marginal utility if $\Upsilon = 0.5$. Hence, in this model, consumers do not have an a priori specific preference for one type of good over the other. We prefer to opt for this assumption in that the choice of a specific functional form might affect final allocations[17], without being really backed by sufficiently strong empirical evidence. Existing studies, indeed, are not yet conclusive on whether consumers actually prefer sustainable goods over others. For example, some studies suggest that environmental concerns are important in consumption choices [18] while other argue that consumers are rather driven by factors like brand and prices when taking their decisions (see, among others, Hardisty et al. (2016) and de Vicente Bittar (2018)). Setting similar preferences for brown and green goods is a more conservative choice to prevent that a results might be driven by specific functional forms that favour green production over brown.[19]

3.1.2 Producers

In each country there are two *continua* of firms: brown and green. Both firms use the same labor and produce undifferentiated intermediate goods. The only difference is in the production technology, as brown firms use a standard and polluting technology which entails a negative externality (emissions) for the environment which in turns generates aggregate costs. Examples of these types of costs are the effects of climate-induced disasters like floods, heavy rainfalls and hurricanes, as well as the health costs of diseases; Hsiang et al. (2017) provides a quantification of several of these risks for the US. Based on what discussed in Section 2, we also assume that brown production relies on more standard and trustworthy production technologies, thus

[15]The aggregators for government spending and investments as well as CES demand functions are derived in Appendix B.3.

[16]Similarly to ω for home and foreign goods, Υ pins down the steady state ratio of brown to green goods. In other terms, captures households preferences for green or brown goods; we set it to 0.5 to allow preference to be identical between the two.

[17]It would be relatively easy to assume that consumers derive higher utility from green consumption. As a result, consumers would rebalance strongly towards green goods after a shock, hence reinforcing our conclusions.

[18]See also Hojnik et al. (2019) for a review of the literature and the different conclusions.

[19]Stronger preferences for green would just make the rebalancing between brown and green production stronger; in that case our quantitative results can be considered as a lower bound.

making the marginal productivity of capital in the brown sector higher. Conversely, green production does not generate the negative externality, but is intrinsically more expensive and, hence, capital there invested is less productive[20]. These assumptions model the fundamental trade-off between different production technologies: brown goods rely on existing and refined technologies hence are cheaper but their production generate aggregate losses; on the contrary, green goods require more investment hence are more expensive, but have no detrimental effects on the society as a whole. Differences in returns is also necessary to justify the co-existence of the two types of firms and avoid corner solutions.

Specifically, we assume that brown goods are produced with the technology[21]:

$$X_t^b = [1 - \mathcal{L}(x_t)]\, A_t (K_t^b)^{\alpha} (L_t^b)^{1-\alpha} \tag{3.4}$$

with A being a TFP shock and $\mathcal{L}(x_t)$ the loss in production derived from the stock of emissions (externality) in the economy, x_t. $\mathcal{L}(\cdot)$ is a function that quantifies the costs of the current stock of emissions in terms of production losses. Following Heutel (2012), we assume that x_t evolves according to an autoregressive process:

$$x_t = \rho^x x_{t-1} + \mathcal{A}_t e_t + \mathcal{A}_t^* e_t^* \tag{3.5}$$

where ρ^x is the decay rate of emissions, which is inversely related to the amount of the externalities that the environment is able to absorb in each period. Empirical evidences suggest that the half-life of emission ranges between 139 and 83 years, implying a relatively high value of ρ^x (0.9979). We introduce also two shocks to emissions, $\mathcal{A}$ and $\mathcal{A}^*$, which proxy for sources of emissions outside the production of goods, including emissions from other countries[22].

e and e^* are the emissions by the domestic and foreign economy respectively. They are a function of the production of brown goods:

$$e_t = (X_t^b)^{1-\gamma} \tag{3.6a}$$

$$e_t^* = (X_t^{*,b})^{1-\gamma}. \tag{3.6b}$$

[20] In Section 4, we estimate the model with different production technologies and we test whether productivity of capital is indeed different.

[21] We drop the index k for presentation convenience. See Appendix B.2 for the complete problem.

[22] On an empirical ground, the addition of this shock helps match the data with model-generated series. For example, without this shock it would be harder to disentangle TFP shocks as emissions would be collinear with GDP production. Moreover, historical data, indeed, show that the production sector does not account for the entire amount of emissions produced. This issue can be compared to that of measurement errors in time-series.

Notably, in this framework, emissions generated in one country produce a negative externality also in the other. Because the stock of emission x is very persistent, only coordinated reductions across countries are effective in reducing the cost of climate events $\mathcal{L}(x)$.

The production of green goods, instead, does not generate externalities but is equally affected by the level of emissions:

$$X_t^g = [1 - \mathcal{L}(x)]\, A_t (K_t^g)^{\alpha\bar{\psi}} (L_t^g)^{(1-\alpha)}. \tag{3.7}$$

$\bar{\psi}$ is a scalar that defines the marginal productivity of capital of green to brown firms. Notice that green firm production does not generate social costs, but might be less efficient in terms of output per unit of capital. This trade-off *de facto* allows brown and green productions to cohexist in the model. The loss function $\mathcal{L}(\cdot)$ takes the following form:

$$\mathcal{L}(x_t) = d_1 x_t^{d_2}, \tag{3.8}$$

which implies that the externality produced in the brown sector of one country affects both the domestic green sector as well as the overall foreign production. In Section 4, d_1 and d_2 in Equation (3.8) will be estimated using prior values derived from Heutel (2012) and Golosov et al. (2014). In its original formulation in the DICE model, see Nordhaus (2017), the loss function has a linear-quadratic specification and takes as argument the temperature level, which is itself a function of radiative forcings of greenhouse gases that depend of the total stock of emissions. Because temperature levels do not need to appear explicitly in this model, our loss function relates directly emissions with GDP losses, thus keeping the model more tractable. In other terms, it captures the reduced-form relation between higher emissions and lower output caused by more severe climatic events. Because there are no established sources to calibrate its coefficients, we estimate them based on GDP and emission data.[23].

The introduction of $\mathcal{L}(x_t)$ introduces a feedback-loop in production, which is the key mechanism of climate macro-models. Any policy that increases output also rises the level of emissions thus generating more GDP losses and dampening the effects of the policy. The resulting equilibrium

[23] Golosov et al. (2014) use an exponential specification of the loss function: $1-\mathcal{L}(x_t) = \exp(-\gamma_t(x_t-\bar{x}))$, where $\bar{x}$ represents the pre-industrial emission average. Heutel (2012), on the other hand, adopts a linear-quadratic specification: $\mathcal{L}(x_t) = d_0 + d_1 x_t + d_2 (x_t)^2$ derived from the DICE model. As highlighted above, however, the linear-quadratic specification of the loss function in Nordhaus (2017) has temperature as input. In this setting, the former alternative seems preferable for practical reasons. Moreover, the form proposed by Heutel (2012) indeed generates a trend in output, to the contrary of all the other variables in the model. This would represent a departure from the standard specifications of New-Keynesian open economy models and would lead to issues at the estimation stage. Our formulation of the loss function is a generalization of those proposed by Heutel (2012) and Golosov et al. (2014), as the estimation of parameters would determine whether the linear or exponential component would dominate in generating the externality, in the same spirit of the empirical analyses presented in Section 2.

is therefore the result of a combination of the positive effects of the policy and of the negative effects of rising emission costs. In quantitative terms, this implies that a model neglecting the cost of climate events might overestimate the effects of shocks. Appendix C.1 shows how the deterministic steady state and first order impulse responses change when emission externalities are included in a version of the model with symmetric calibration for the two countries. Appendix C.2 shows, for the same calibration of structural parameters, how optimal monetary policy changes when the cost of emissions increases. Notably, higher emissions entail a stronger response to output, because when climate costs rise, output becomes less volatile in both economies, see Table C.2. This is a direct consequence of the negative feedback loop between emissions and output (i.e. higher output increases emissions which in turns reduce output in the future). When climate costs rise above a specific threshold, responding positively to output becomes optimal.

4 Estimation

We estimate the model with euro area and U.S. data using 12 macroeconomic time series: real GDP (y^o), inflation (π^o), real aggregate consumption (c^o), labor supply (l^o), the policy rate (r^o) and the stock of CO_2 emissions (e^o).

With the exclusion of emissions and the policy rate variables are defined as in Smets and Wouters (2007). Specifically, variables are expressed in per-capita growth rates, de-trended and seasonally adjusted. Emissions are also expressed in per-capita terms and are de-trended. When the effective lower bound is binding we use shadow rates instead of the short-term rate; interest rates and inflation rates are defined in quarterly terms. Variables are related to the model through the following observable equations in the domestic and foreign economies[24]:

$$\begin{bmatrix} y_t^o \\ \pi_t^o \\ c_t^o \\ l_t^o \\ r_t^o \\ e_t^o \end{bmatrix} = \begin{bmatrix} \log(Y_t) - \log(Y_t) \\ \pi_t - 1 \\ \log(C_t) - \log(C_{t-1}) \\ \log(L_t) - \log(L_{t-1}) \\ r_t - \frac{1}{\beta} \\ \log(\mathcal{A}_t e_t) - \log(\mathcal{A}_{t-1} e_{t-1}) \end{bmatrix}. \tag{4.1}$$

[24]Refer to Appendix A for information on data and sources.

4.1 Calibrated parameters and priors

Following Smets and Wouters (2007) we estimate only a subset of the structural parameters[25], while we calibrate the rest by following the relevant literature. The habit formation parameter h is set to 0.75 as in Eichenbaum et al. (2021). As explained in Campbell and Cochrane (2000) and Benmir et al. (2020), habit formation plays an importat in DSGE models, in that it generates inter-temporal correlation for consumption. Such feature allows for credible model-based risk-premia and stochastic discount factors, which in turn determine to what extent households are willing to tolerate volatility in asset prices before substituting across them. Since in our model households choose between brown and green assets, it is indeed crucial to generate reasonable dynamics of risk-premia.

ρ^x, the parameter measuring the persistence of emissions, is calibrated to 0.9979, in line with both empirical evidence and other models featuring emission externalities (Nordhaus (2008), Heutel (2012) and Benmir et al. (2020)). As in Eichenbaum et al. (2021) α is set to 0.3, the home bias, ω, to 0.9, the elasticities ρ and ν to 1/3 and 6 respectively, φ is set to 1 and we use log-utilities. Finally, we set ψ, the parameter driving the wedge between steady state returns on brown and green assets, by using Greenhouse Gas Protocol (GHG) scores for equities[26]. Specifically, we calibrate ψ to match the difference in average returns between companies in the top and bottom 10% of the distribution of emissions, namely 0.98 in the euro area and 0.93 in the US (Table D.5). Other calibrated parameters are reported in Table D.6.

As to the estimated parameters, we use the standard prior specifications adopted by Eichenbaum et al. (2021). Therefore, the shocks' standard deviations are set to follow an inverse gamma distribution with mean of 0.01 and standard deviation of 2. The prior distribution of the autoregressive component of shocks, on the other hand, is a gamma with mean equal to 0.5 and standard deviation equal to 0.2. The persistence of the Taylor rule, γ_r, follows a beta distribution with mean 0.75 and standard deviation 0.1. The inflation and output coefficients, θ_π and θ_y, have a normal distribution with means 1.2 and 0.6 and standard deviations 0.2 and 0.1 respectively. The Calvo pricing parameter ξ follows a beta distribution with mean 0.75 and standard deviation 0.1.

For the climate-specific parameters no standard prior choice is indicated by the literature. In our model the normal prior mean of γ, which determines the link between new emissions and current brown production, is set equal to 0.304 as in Nordhaus (2008), whereas its standard

[25] A similar approach is also used by Gerali et al. (2010), Angelini et al. (2014) and Eichenbaum et al. (2021).

[26] GHG scores are produced by the World Resources Institute based on reported greenhouse gas emissions by company. It is a standard source to get information about the environmental footprint of private and public entities. Refer to https://ghgprotocol.org/ for additional information.

deviation is equal to 0.1. d_1 and d_2, which fall in the interval $[0, \infty)$ and determine how much the stock of emissions is detrimental to production, follow an inverse gamma prior distribution with mean and standard deviation both equal to 0.01.

4.2 Posterior

Posterior parameters are reported in Table D.7. Standard New-Keynesian parameters are broadly in line with Eichenbaum et al. (2021), the slight discrepancies deriving essentially from the differences in countries, the estimation samples considered and obviously the introduction of climate externalities.

As to the climate parameters, the posterior estimates of γ are smaller for the US than for the euro area. In the US, brown production generates more externalities per unit of output with respect to what found by Nordhaus (2008). This might be due to the fact that our sample covers the period between 2000 and 2018, while Nordhaus (2008) uses pre-2000 data only, thus excluding years where emissions have increased the most[27]. On the other hand, the production technology generates less emissions in the euro area. These results are confirmed by the parameter estimates for the cost of emissions, d_1 and d_2, which are found to be larger in the US than in the euro area. This might also result from the different climate policies implemented on the two sides of the the Atlantic. In regard to the impact of climate change, our estimates suggest that the US economy is more affected than the euro area, with a loss of steady state output of 1.2% and 0.4% respectively. These numbers are in the range of the losses for the US economy in the early 90s as estimated by, among others, Nordhaus (1994) and Tol (1995). In more recent studies the cost of emissions in the US lies between 1.4% (Hsiang et al. (2017)) and 32 dollars per ton or 3.2% of GDP (Nordhaus (2017))[28]. State-of-the-art structural models with emissions externalities generally entail lower impacts of emissions on GDP. Heutel (2012) and Ferrari and Nispi Landi (2020), for instance, estimate steady state losses of 0.26% and 0.7% of GDP respectively.

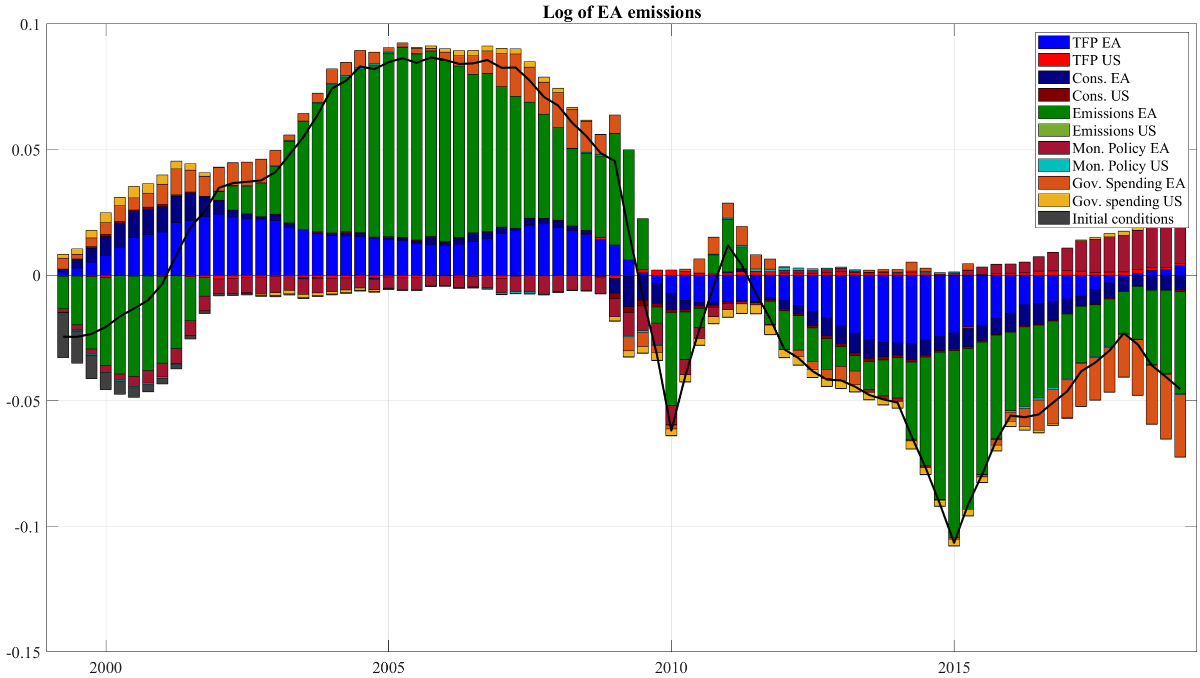
Figure 6 and Figure 7 plot the historical decomposition of emission levels for the US and the euro area. The decomposition shows a marked reduction of the contribution of emission shocks, $\mathcal{A}$ and $\mathcal{A}^*$, which in this empirical specification capture emission spillovers from third countries over the two economies. Such drop has started with the 2008-2009 financial crisis, and has brought about an average contraction in per-capita emissions by around 6.2% in the euro area and 4% in the US over the period 2009-2018. Generally speaking, the contribution of TFP

[27] See Canadell et al. (2007) and the related Reuters articles: World CO_2 emissions speed up since 2000 (2007) and World CO_2 output to rise 59 pct by 2030: U.S (2007).

[28] Refer also to Burke et al. (2015) and Tol (2009) for a comprehensive review of estimates produced by the relevant literature. Generally speaking, emission costs are found to range from 0.4% to 1.9% of GDP.

shocks has turned negative over that period, which suggests improvements in the production technology. On the other hand, while the decrease in the euro area has been more constant since 2010, in the US there have been several ups and downs in the emissions level due to swings in the spillovers from the rest of the world that have been partially offset via changes in the US government spending. However, US emissions have been on the rise again since 2017, following some radical changes in domestic government spending and a reduction in negative emissions shocks from external economies. This has had repercussions on the euro area as well via the positive contribution of US consumption to euro area emissions.

Figure 6: Historical decomposition of log per-capita emissions in the euro area

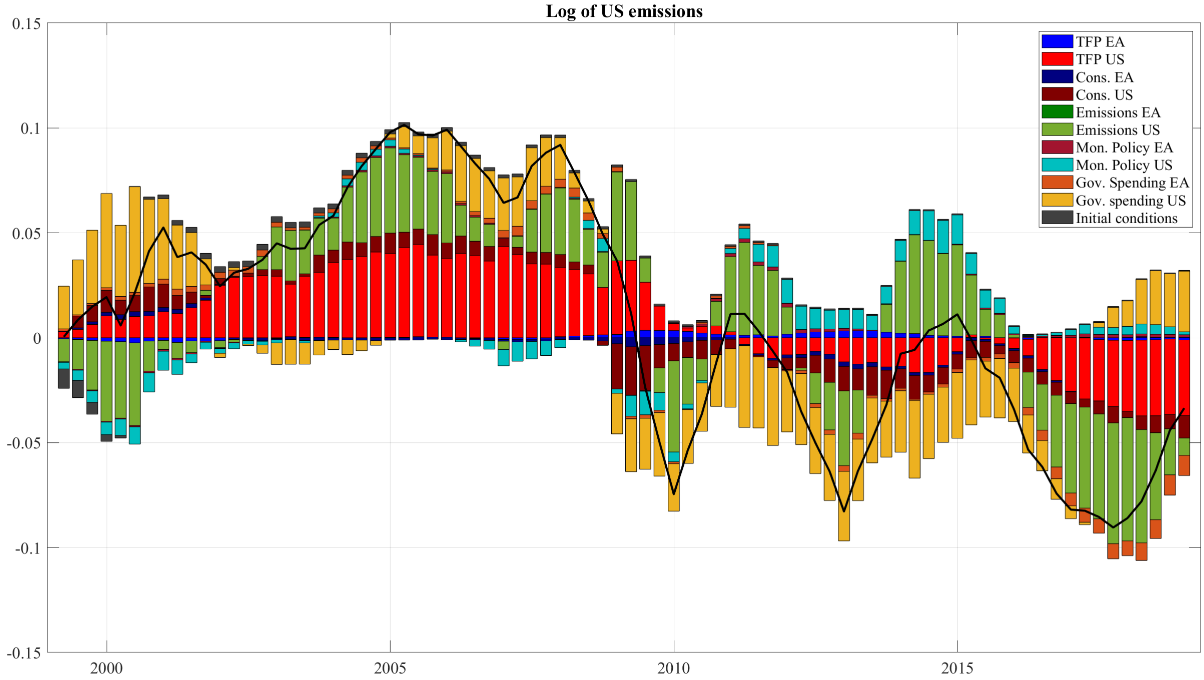


5 Containment policies

In this section, analyze three policy alternatives to tackle emissions containment. First, we list the characteristics of each approach and how we implement it in the model. We then discuss the results produced by the different specifications. For the purpose of the policy exercises, we solve the model at second order with pruning as in Kim et al. (2008) and Andreasen et al. (2018), so to account for volatility effects. Under pruning, the state space representation of the dynamic system approximated via a second-order Taylor expansion is:

$$\begin{aligned} x_{t+1}^s &= h_x x_t^s + \frac{1}{2} H_{xx} \left[x_t^f \otimes x_t^f \right] + \frac{1}{2} h_{\sigma\sigma} \sigma^2 \\ y_t^s &= g_x \left[x_t^s + x_t^f \right] + \frac{1}{2} G_{xx} \left[x_t^f \otimes x_t^f \right] + \frac{1}{2} g_{\sigma\sigma} \sigma^2, \end{aligned} \tag{5.1}$$

Figure 7: Historical decomposition of log per-capita emissions in the US



where x^n and y^n are state and control variables proxied at first ($n = f$) and second ($n = s$) order with pruning. h_x, g_x, H_{xx} and G_{xx} are matrices of first and second order derivatives, while $h_{\sigma\sigma}$ and $g_{\sigma\sigma}$ are matrices of loadings for the volatility of shocks (σ^2).

The second-order accurate mean of states, x^s, is:

$$E\left[x_t^s\right] = (I - h_x)^{-1}\left(\frac{1}{2}H_{xx}\left[x_t^f \otimes x_t^f\right] + \frac{1}{2}h_{\sigma\sigma}\sigma^2\right) \tag{5.2}$$

while the second-order mean of controls can be derived from Equation (5.1). Equation (5.2) shows that the mean of variables at second order depends on two components: i) the volatility of shock processes, with loadings given by the $h_{\sigma\sigma}$ matrix; ii) the deep parameters and the deterministic steady state of the model, i.e. the approximation point, that define the matrices of derivatives. In this solution, the importance of volatility for the second-order mean of endogenous variables vis-à-vis the deterministic steady state depends on the matrices of derivatives. Variables for which volatility does not matter would have very low loadings in the matrices $h_{\sigma\sigma}$, H_{xx}, $g_{\sigma\sigma}$ and G_{xx}. Therefore, changes in policies that work mainly through a reduction in volatility, such as monetary policy, would have little effect on their second-order mean[29]. On the other hand, the second-order mean of the same variables could be more affected by changes in the deterministic steady state or the deep parameters of the model. This result holds in any higher-order approximation because all higher-order moments depend on volatility through the

[29]As an example, the TFP process depends only on its lag and all loadings in $h_{\sigma\sigma}$, H_{xx}, $g_{\sigma\sigma}$ and G_{xx} are 0.

state space matrices of the approximated model, which define first- and second-order derivatives[30]. The implications of this analytical results are straightforward: policies can affect both the long-run growth trend of the economy, in these models captured by the deterministic steady state, or the volatility of macro variables around that trend. Different variables might depend more from one of those two components and, as a consequence, be more or less impacted by specific policy changes.

In this model, for example, the stock of CO_2 emissions, x_t, tends to be extremely persistent, implying that it might respond more to steady-state effects rather than volatility effects.

Before proceeding with the analysis of the single policies, we deem it dutiful to formally defined our evaluation approach. We rank the different configurations of containment policies according to the associated households' welfare, which can reflect the effect of policies on the present and future discounted utility of households as well as the impact on prices, income, employment and wealth. For example, policies that cut emissions today might make increase prices or reduce production, thus affecting households' welfare. Because in macro models, households can be considered as the "principals" of policy-makers, their welfare is a good proxy to determine whether a policy is incentive-compatible, that's to say whether it makes households better off or not. This metric, however, presents the drawback of not accounting for the very long-term more severe effects of climate change. At the same time, these latter ones, once properly considered, would make any climate policy welfare-improving.

Consider, as an example, the case where households seriously discount the possibility that at some point in the future the world will become unsuited for human life. That scenario, when and if realized, would generate a welfare loss of $-\infty$. Any policy either reducing today the likelihood of such an event or simply postponing it to a more distant future would improve welfare by $+\infty$, regardless of the discount factor. In light of this, a welfare-based assessment of containment policies is suited to capture their short-term costs only. Nonetheless, our analysis is useful to well outline the political-economy problem that is currently being faced by decision-makers[31]. Turning to the objectives of the climate policy, we set it to a reduction of emissions by 50% in line with what agreed on international fora, see IMF (2019). Different policies are ranked by their ability to reach that objective and by the implied welfare losses. A policy is *incentive compatible* if it does not reduce welfare, so that agents would be willing to implement

[30] For example the third-order mean of state variables is:

$$E\left[x_t^{rd}\right] = (I - h_x)^{-1}\left(H_{xx}\left[x_t^s \otimes x_t^f\right] + \frac{1}{6}H_{xxx}\left[x_t^f \otimes x_t^f \otimes x_t^f\right] + \frac{1}{6}h_{\sigma\sigma\sigma}\sigma^3\right).$$

Refer also to Andreasen et al. (2018).

[31] In this regard, if agents in our model would actually discount the possibility of a future, yet remote, catastrophic climate disaster, any mitigation policy would be welfare-enhancing and readily implemented *today*.

it. Welfare is reported in terms of consumption equivalence relative to the baseline model.

1. Monetary policy It has been recently debated that monetary policy could play a role in fighting the effects of climate change (NGFS (2020), Villeroy de Galhau (2021)). We evaluate the effectiveness of monetary policy intervention by comparing optimal policy with and without climate costs and by allowing the central bank to directly target the level of emissions. Since our model is estimated using the shadow rate, r, such indicator can be interpreted as a broad measure of the monetary policy stance, thus allowing the policy rate to go below zero and approximating unconventional policies. For an extensive discussion of the impact of green QE see Ferrari and Nispi Landi (2020).

2. Domestic emission tax and abatement The environmental externality is a textbook example of a market failure: brown firms do not bear any direct cost from the externality and, as a consequence, they do not take it into account when producing. For this reason, in the baseline model we do not include any abatement policy because brown firms won't have any incentive to reduce emissions.

The market failure opens up to the possibility for a welfare-enhancing public intervention in the form of an emission tax. Specifically, a tax proportional to the amount of emissions produced would reduce returns on capital on brown production, thus changing the equilibrium allocation of capital between the brown and the green sectors. Moreover, the tax would make the emissions become a cost for brown firms, which would then have the incentive to invest in abatement technology. As in standard climate models, it is assumed that the abated share of production by brown firms does not pollute.

Specifically we assume that the government could introduce an emission tax, τ^b, which is proportional to the volume of emissions that each firm produces. In addition, brown firms can decide to abate a fraction $\mu \in [0,1]$ of emissions produced. The abatement technology has a cost $\theta_1 \mu^{\theta_2}$, where θ_2 is the curvature of abatement cost function and θ_1 pins down the steady-state[32]. This specification is similar to Heutel (2012) and Benmir et al. (2020). Brown firms' profits become:

$$\Pi_t^b(k) = P_t X_t^b(k) - R_t^{k,b}(k) K_t^b(k) - W_t^b(k) L_t^b(k) - \tau_t e_t(k) - \vartheta_1 \mu_t(k)^{\vartheta_2} X_t^b(k) \tag{5.3}$$

where:

$$e_t(k) = (1-\mu_t)\left[(1-\mathcal{L}(x_t)) A_t K_t^b(k)^\alpha L_t^b(k)^{1-\alpha}\right]^{1-\gamma} \tag{5.4}$$

[32]Refer to Gillingham and Stock (2018) for a discussion on the estimation of abatement costs.

The abatement technology reduces emissions and this, in turn, decreases the amount of taxes that firms have to pay, thus making emissions endogenous to the firms' problem. The new first order conditions are:

$$R^{k,b}t(k) = \alpha MC_t(k)\frac{X_t^b(k)}{K_t^b(k)} - \tau_t(1-\mu_t)\alpha\frac{X_t^b(k)}{K_t^b(k)} \tag{5.5a}$$

$$w_t(k) = (1-\alpha)MC_t(k)\frac{X_t^b(k)}{L_t^b(k)} - \tau_t(1-\mu_t)(1-\alpha)\frac{X_t^b(k)}{L_t^b(k)} \tag{5.5b}$$

$$\mu_t = \left(\frac{\tau_t(X_t^b(k))^{1-\gamma}}{\theta_1\theta_2}\right)^{\frac{1-\theta_2}{\theta_2}} \tag{5.5c}$$

Equation (5.5c) defines the share of abated emissions as a function of their cost. When $\tau = 0$, as in the baseline model, firms choose not to abate; moreover the marginal productivity of capital and wages in brown firms is decreased by a factor proportional to the emission tax. In other terms, firms internalise the costs of new emissions (net of the abatement) when producing. The parameters θ_1 and θ_2, which determine the marginal cost of the abatement technology, are set equal to 0.0560 and 2.8 respectively, as in Heutel (2012) and Benmir et al. (2020). These values are derived from the empirical estimates of Nordhaus (2008).

We assume that the government could use the tax revenues either to finance its budget, thus reducing lump-sum taxes paid by the households, or to subsidize green production. Intuitively, as the green production technology is less efficient than the brown one, the emission tax could act as a transfer to equalize returns on capital across the two sectors. These transfers can be for instance interpreted in the spirit of the "research subsidies" discussed by Acemoglu et al. (2016). The subsidy per unit of green production is constrained by the total tax revenues:

$$\tau_t e_t = \bar{s}_t X_t^g \tag{5.6}$$

where the tax rate depends on a steady state parameter: $\tau_t = \tau_{ss}$[33].

We consider three possible setups for the implementation of the environmental tax: i) each country *individually* levies a tax leading to the reduction of its own emissions by 50%; ii) both countries engage in a non-cooperative game, whereby they strategically set the domestic tax by taking as given the other country's decision; iii) the two countries settle for a cooperative equilibrium by jointly setting the tax level to reduce global emissions by 50%. In the Nash equilibrium of the non-cooperative game we compute the optimal response function of country

[33] In Appendix C.4 we explore richer tax policy rules which respond to output and asset prices.

i to the policy of the other country as:

$$\max_{\tau_i} E(W_{i,t})|(\tau_{j\neq i}, \Omega_i), \quad i,j \in \{EA, US\} \tag{5.7}$$

where $\tau_{j\neq i}$ is the taxation adopted by the other country and Ω_i includes country i's constraints given by the structural equations of the model. Welfare is defined in terms of consumption equivalent, i.e. $W_{i,t} = U_{i,t} + \beta E_t(W_{i,t+1})$, while $E(W_{i,t})$ is the unconditional mean, which might differ from the deterministic steady state because of uncertainty. The problem described by Equation (5.7) is a game: for each level of the opponent's tax rate, $\tau_{j\neq i}$, the domestic country chooses the optimal rate, τ_i, that maximize its welfare given the structure of the economy, Ω_i. The sequence of τ_i for each possible value of $\tau_{j\neq i}$ is country i's reaction function. The Nash equilibrium(a) of the game is (are) given by the intersections of the reaction functions of the two countries. For each case we analyse the effectiveness of taxes in reducing emissions, its welfare implications and the compatibility of emissions taxes with country-specific incentives. Additional details on the game are provided in Appendix C.5 and Ferrari and Pagliari (2021).

3. Import tariff on foreign brown goods Countries might rely on tariffs[34] because taxing domestic brown goods reduces total emissions, but at the same time might also harm the economy. As an alternative, the government could decide to "export" the cost of climate policies by taxing imports of brown goods from the foreign economy. There are several reasons why countries might consider imposing such tariffs. Firstly, they would reduce trade in brown goods, thus arguably inducing the targeted country to lower its emissions. Secondly, the tariff-imposing country might improve its balance of payment and boost its exports. Finally, tariffs could also be used as a punishment strategy if one country does not comply with the emission reduction agreements.

However, there are also reasons why taxing brown imports might not be optimal. It is for instance unclear whether the elasticity of emissions to import tariffs is positive. Import tariffs might indeed provide domestic brown firms with a competitive advantage, which could eventually lead to an increase in domestic brown production and, hence, emissions. Similar undesired effects might also take place in the foreign country, with brown production increasing in response to the tariff. The relative strength of these channels depends on consumption elasticities in both countries and on the elasticity of import and exports to the exchange rate.

In addition, the partner country could retaliate by starting a "trade war" whose costs might

[34] Brown tariffs have been explored already in the context of gravity models as in Larch and Wanner (2017); in this frameworks, however, exchange rate and monetary policy reactions might trigger different general equilibrium dynamics.

be significant (Lindé and Pescatori (2019)). Lastly, tariffs interfere with optimal price setting, thus resulting in inefficient allocations. Most importantly, when compared to *domestic* taxes, tariffs do not provide brown firms with the incentive to abate emissions, which is a key factor for making climate policies effective. Therefore, welfare losses stemming from tariffs might all in all exceed their benefits. Finally, the exchange rate might move to compensate the tariff, thus leaving equilibrium allocations unaffected[35].

In our framework, we implement tariffs as in Lindé and Pescatori (2019) and Jacquinot et al. (2020). Specifically, the price of imported brown goods in the home country (euro area) expressed in the local currency units becomes:

$$\frac{P_{F,t}^{*,\bar{b}}(k)}{P_t} = RER_t \frac{\mathcal{K}_{F,t}^{*,b}}{\mathcal{F}_{F,t}^{*b}} \tau^{Imp} \tag{5.8}$$

where $P_{F,t}^{*,\bar{b}}(k)$ is the price of imported goods in the home country, $\mathcal{K}_{F,t}^{b}$ and $\mathcal{F}_{F,t}^{*b}$ are the two recursive terms that define the optimal price setting for exporters, and τ^{Imp} is the import tariff[36]. We do not consider the case of an export subsidy, as the Lerner symmetry applies to this model[37]. We also do not consider the strategic interactions between different carbon tariffs as the topic has been already extensively analyzed by Larch and Wanner (2017), Böhringer et al. (2016) and Nordhaus (2015) under different modelling assumptions.

5.1 Monetary policy

In this section we consider the optimal monetary policy problem of the social planner that optimizes the parameters of the policy rule to maximize households welfare[38]:

$$\ln R_t = \gamma_r \ln R_{t-1} + (1-\gamma_r)\left[\ln R_{ss} + \theta_\pi \ln \pi_t + \theta_Y \left(\ln Y_t - \ln Y_{t-1}\right)\right] + \varepsilon_t^r. \tag{5.9}$$

We consider both a decentralized equilibrium, with each country optimizing domestic welfare only, and a cooperative equilibrium, where the objective function is given by a weighted average of both countries' welfare. Optimal policy parameters and stochastic steady state of variables under the optimal policy are reported in Table 4 for the two versions of the model with and without climate externalities[39]. In our model, the optimal policy implies a strong reaction to

[35]See Eichengreen (2017) for a discussion of the exchange rate reaction to the Sino-American trade war.

[36]The full problem of exporters is solved in Appendix B.4 from the home country perspective. The equations for the foreign country are symmetric with foreign-country variables denoted by *.

[37]See McKinnon (1966).

[38]Welfare is defined using the recursive representation $W_t = U_t + \beta E_t(W_{t+1})$.

[39]The model is solved at second order to compute second-order accurate welfare values. The stochastic steady state at second order differs from the deterministic steady state because of the contribution of future shocks on the asymptotic mean of these variables. Specifically, we apply the algorithm of Born and Pfeifer (2020).

Table 4: Optimal monetary policy

	Climate externality			No climate externality		
	EA	US	Global	EA	US	Global
γ^{EA}	0.35	-	0.57	0.35	-	0.59
θ_{π}^{EA}	3.50	-	3.37	3.50	-	3.48
θ_{y}^{EA}	0.00	-	0.14	0.00	-	0.12
γ^{US}	-	0.64	0.69	-	0.64	0.69
θ_{π}^{US}	-	3.50	3.50	-	3.49	3.50
θ_{y}^{US}	-	0.00	0.00	-	0.00	0.00
$E(W_{EA})$	1.68	0.03	1.66	2.37	2.42	2.39
$E(W_{US})$	-0.05	0.52	0.45	1.80	2.30	2.29
$E(W_{Global})$	0.79	0.28	1.04	2.07	2.36	2.34
$E(e_{EA})$	0.18	0.01	0.17	0.48	0.48	0.48
$E(e_{US})$	0.01	0.10	0.10	1.25	1.34	1.34
$E(x)$	0.09	0.06	0.13	0.90	0.95	0.95

Notes: optimal coefficients of the monetary policy rule for each country in the cooperative and non-cooperative equilibria and in presence or absence of climate externalities in the model. Welfare is defined in recursive form as $W_t = U_t + \beta E_t(W_{t+1})$ and global welfare is the weighted average of welfare in the two countries: $W_{Global,t} = 0.5W_{EA,t} + 0.5W_{US,t}$. Welfare and emissions (e_{EA}, e_{US}, x) are expressed in percent deviation from the estimated model. The second order stochastic steady state of variables ($E(\bullet)$) differs from the deterministic steady state because of the contribution of future shocks on their asymptotic mean.

inflation and a somewhat limited response to output both in the decentralized and cooperative equilibria. Notably, while optimal policy coefficients are essentially identical across the two specifications of the model, welfare values are markedly different. In the model with climate externalities, the optimal policy delivers significantly lower welfare gains compared to the baseline specification and spillovers are also reduced. For instance, in the case of the US, under optimal monetary policy and absent the emissions externalities, welfare increases by 2.3% domestically and by 1.8% in the euro area relative to the estimated model. On the other hand, in presence of climate costs, gains from the optimal policy in the US drop to 0.5% and there are no positive spillovers to the euro area.

Under the optimal policy, indeed, households consume more compared to the baseline specification. As demand increases, overall production, using both technologies, needs to expand to catch up, thus generating more emissions and increasing the aggregate loss function ($\mathcal{L}(x)$). This in turn triggers a feedback loop across demand, production and climate costs. As output expands, emissions increase, which leads to lower revenues, households' income, wages and, eventually, a lower steady state welfare. This mechanism is evident when comparing the levels of emissions in Table 4. When there are no emissions costs, the optimal policy increases the levels of emissions by 0.5% up to 1.3%. When the cost of climate externalities ($\mathcal{L}(x)$) is included into the model, on the other hand, emissions increase much less because of the feedback loop just described. The presence of emission costs prevents the economy from taking full advantage of the optimal monetary policy and reaching full potential, because any additional unit of output generates also costs for the economy. Hence, monetary policy appears to be significantly less effective in the model with emission externalities. In Appendix C.3 we investigate whether the central bank can reach a significant reduction of emissions through monetary policy only. Results reported in the Appendix suggest that when the monetary authority targets emissions it generates welfare losses relative to the baseline framework and does not significantly reduces new emissions. This result holds also when the central bank targets a weighed average of welfare and climate objectives.

5.2 Fiscal policy

The second policy under scrutiny is a government tax on brown production, which is proportional to the amount of emissions as in Equation (5.5c)[40]. Table 6 shows the optimal taxation level when governments seek to optimize households' welfare both in isolation and in cooperation (i.e. jointly setting the taxation rate and targeting global welfare) without strategic interactions.[41] In this exercise the tax level is constrained to be > 0 to verify whether a positive taxation of emissions is indeed incentive compatible in the model. Under a welfare objective, the "optimal" level of taxation is not high enough to achieve the necessary reduction of emissions, which decrease by about 20%, independently on the use of the tax's revenues. Most importantly, the climate tax induces welfare losses in both countries relative to the baseline model without taxation. As such, countries will choose not to impose these taxes in the equilibrium because of welfare costs and, therefore, no reduction of emission is achieved.

We now consider a more complex fiscal policy framework where i) governments seek to reduce emissions to a level consistent with the Paris 2 agreement (i.e. 50% below current levels), independently of welfare losses, and ii) countries engage in a non-cooperative Nash game when setting the environmental tax.

Table 6 reports the optimal tax rate for each country, together with the reduction in welfare and emissions relative to the baseline model without taxation. We consider three scenarios: i) each country implements the tax independently to reach the climate objective (Columns (1), (2), (5) and (6)); ii) both countries enter a *non-cooperative* game determining the domestic tax

[40] With the tax, brown firms internalize the costs of emissions and have the incentive to invest in the abatement technology. We assume that the government can use tax revenues either to finance its budget, thus lowering households taxation, or to subsidize green firms. In the latter case the subsidy would be proportional to the total amount of revenues, which in turn would reduce the gap between the productivity of brown and green capital and increase the competitiveness of green firms. Appendix C.4 we present similar results with a tax which is allowed to vary along the business cycle.

[41] Global welfare is defined as the weighted average of welfare in both countries: $W_{Global,t} = 0.5W_{EA,t} + 0.5W_{US,t}$.

Table 5: Optimal fiscal policy

	Transfer to households			Transfer to green firms		
	EA	US	Coop.	EA	US	Coop.
τ_{EA}	0.01	-	0.01	0.01	-	0.01
τ_{US}	-	0.01	0.01	-	0.01	0.01
$E(W_{EA})$	-0.18	0.00	-0.18	-0.28	0.00	-0.28
$E(W_{US})$	0.01	-0.16	-0.15	0.01	-0.28	-0.28
$E(W_{Global})$	-0.08	-0.08	-0.17	-0.13	-0.15	-0.28
$E(e_{EA})$	-17.26	0.00	-17.26	-17.26	0.00	-17.26
$E(e_{EA})$	0.00	-19.32	-19.32	0.00	-19.32	-19.32
$E(e_{Global})$	-7.80	-10.59	-18.39	-7.80	-10.59	-18.39
Tax EA (% of GDP)	0.27	0.00	0.27	0.27	0.00	0.27
Tax US (% of GDP)	0.00	0.33	0.33	0.00	0.33	0.33

Notes: τ_{ss}: optimal tax rate under the different policy setups. Welfare is defined in recursive form as $W_t = U_t + \beta E_t(W_{t+1})$ and global welfare is the weighted average of welfare in the two countries: $W_{Global,t} = 0.5W_{EA,t} + 0.5W_{US,t}$. Welfare and emissions (e_{EA}, e_{US}, e_{Global}) are expressed in percent deviation from the stochastic steady state of the estimated model. The second order stochastic steady state of variables ($E(\bullet)$) differs from the deterministic steady state because of the contribution of future shocks on their asymptotic mean.

rate as best response to the choice of the other country (Columns (3) and (7))[42]; iii) the two country *jointly* set taxation to reduce global emissions (Columns (4) and (8)). Results suggest that fiscal policy is able to reach the objective with a tax of between 1.1% and 1.2% of GDP. The total revenues from the tax are generally larger in the US, where the economy is more polluting and bears higher costs from climate externalities.

Generally speaking, policies implemented under scenario i) are not sufficient to reduce global emissions to the target. Moreover, they generate domestic welfare losses which are larger the lower the current level of emissions. In other terms, countries whose level of emissions, and the associated cost, is lower have less to gain from a further reduction in emissions. In the case of the euro area, for instance, further reducing emissions would move the economy relatively farther away from the efficiency production frontier, thus generating bigger welfare losses. In the US, on the other hand, as current emissions and the associated costs are higher, a reduction in pollution would generate sizeable output gains, which would lead to more contained welfare costs. This particular policy setup is therefore not incentive-compatible because it reduces the welfare of the country imposing the tax while the other country might obtain welfare gains by increasing the exports of brown goods and, consequently, increasing its emissions. A self-oriented government would hence have no immediate incentives to implement a green taxation scheme.

Policies implemented under iii), whether targeting domestic or global emissions, entail lower costs compared to the case where the two countries would both implement their own policies

[42]Results reported in Columns (3) and (7) of Table 6 are the Nash equilibria of a non-cooperative game, as outlined in Appendix C.5.

independently with a similar taxation (ii)). This suggests that countries should prefer to undertake policies aimed at containing emissions in cooperation with their foreign trade partners. However, even in such scenario, climate policies push the economy away from the equilibrium that maximizes firms profits and households utility, thus leading to the materialisation of welfare costs. Therefore, individual countries could be reluctant to implement these policies or to let others take the lead in this regard, thus missing the emissions reduction objective. Table 6 also shows that welfare losses are decreasing in the costs of emissions. In the US, where emissions account for 1.2% of GDP, welfare losses are 50% lower than in the euro area in scenario i) and are almost zero under the cooperation setup. This important result suggests that, as the costs of climate change increase, countries will have stronger incentives to align in tackling climate change. Unfortunately this might happen too late, when climate change consequences are already irreversible. Finally, in the non-cooperative game countries strategically choose the tax level in order to maximise their own welfare, reacting to the choice of the "opponent". The payoffs for this game are reported in Figure 8. In the Nash equilibrium of the game, the US have no incentive to introduce an emission tax while the euro area optimal taxation is less than 50% of what would be needed to bring emissions down to the target. The resulting equilibrium falls significantly short of reaching the objective and generates moderate welfare losses in the US.

If taxes are directly transferred to households (Column (3)), domestic emissions in the euro area rise with the tax, because the increased demand due to the positive fiscal transfer (i.e. the "income effect") is stronger than the negative impact of the rise in the cost of brown goods (i.e. the "substitution effect"). The same does not hold true when tax revenues are used to subsidize green production (Column (7)). In this case, indeed, there is no fiscal transfer to households and the substitution effect between green and brown products prevails. As taxing emissions is costly, countries prefer to wait for the others to take environmental actions, so that they can rip the benefits of reducing both the stock of CO_2 emissions, x, and the externality losses, $\mathcal{L}(x)$, without directly bearing the policy-related welfare costs. In this context, countries best strategy is then to set too low a level of domestic taxation.

A country's incentive to commit to adequate containment policies is weaker the more relevant is brown production in its economy. As the US brown sector is larger than its euro area equivalent, in the Nash equilibrium the US do not impose any taxation on environmental externalies, whereas euro area's taxation (0.015) is much lower w.r.t. what is needed to attain the reduction target (0.07).

All in all, climate policies are not welfare enhancing because they entail output costs that are reflected in welfare. Consider, for example, a permanent 1% increase in the tax rate of the euro area as reported in Figure E.9. The increase in taxation successfully reduces emissions in the euro area and moves capital away from the brown sector to the green sector. However, green capital does not increase fast enough to compensate the reduction in brown capital, therefore total capital decreases. Moreover, the tax induces an exchange rate appreciation, both in real and nominal terms, which reduces trade. The combined effects of these dynamics leads to a contraction of output in the tax imposing country, reducing consumption and, hence, welfare. Moreover, because the exchange rate appreciates, US exporters export more and brown production expands, leading to an increase in emissions. On a global level, therefore, the effect of domestic taxes is reduced by trade dynamics.

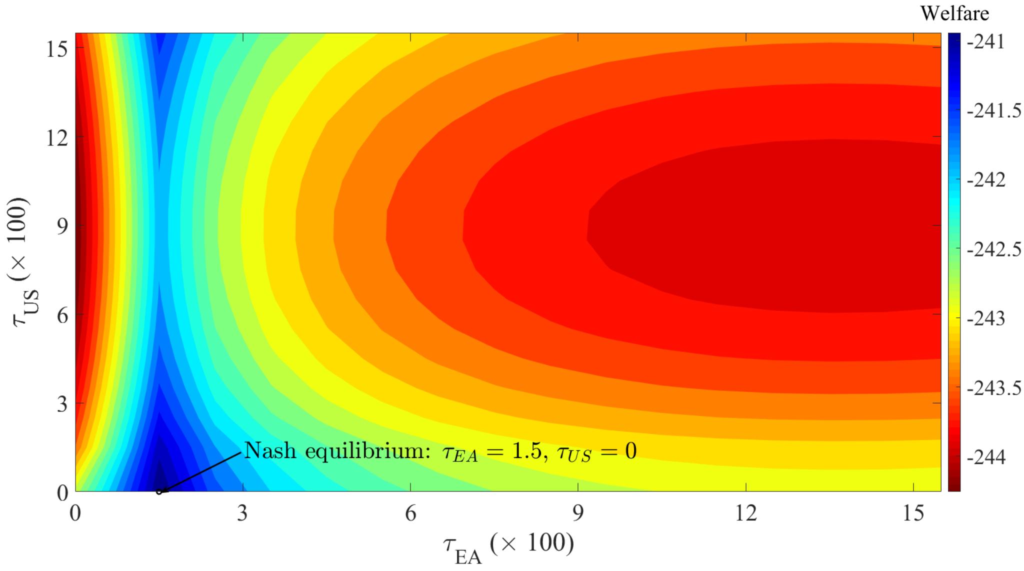
Table 6: Optimal fiscal policy

	Revenues transferred to households				Revenues transferred to green firms			
	EA	US	Nash eq.	Coop.	EA	US	Nash eq.	Coop.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\tau_{ss,EA}$	0.07	0.00	0.02	0.07	0.07	0.00	0.02	0.06
$\tau_{ss,US}$	0.00	0.06	0.00	0.06	0.00	0.06	0.00	0.06
$E(W_{EA})$	-1.23	0.08	1.89	-1.22	-1.67	0.06	0.11	-1.63
$E(W_{US})$	0.46	-0.79	-1.64	-0.04	0.47	-1.11	-1.90	-0.51
$E(W_{Global})$	-0.36	-0.37	0.08	-0.61	-0.57	-0.54	-0.92	-1.06
$E(e_{EA})$	-50.00	-0.15	1.39	-49.82	-50.00	-0.06	-0.82	-45.98
$E(e_{US})$	-0.11	-50.00	-0.19	-50.15	-0.05	-50.00	-0.06	-53.31
$E(e_{Global})$	-22.65	-27.48	0.59	-50.00	-22.62	-27.44	-0.41	-50.00
Tax EA (% GDP)	1.16	0.00	0.49	1.16	1.15	0.00	0.48	1.09
Tax US (% GDP)	0.00	1.19	0.00	1.19	0.00	1.18	0.00	1.23

Notes: τ_{ss}: optimal tax rate under the different policy setups. Welfare is defined in recursive form as $W_t = U_t + \beta E_t(W_{t+1})$ and global welfare is the weighted average of welfare in the two countries: $W_{Global,t} = 0.5W_{EA,t} + 0.5W_{US,t}$. Welfare and emissions (e_{EA}, e_{US}, e_{Global}) are expressed in percent deviation from the stochastic steady state of the estimated model. The second order stochastic steady state of variables ($E(\bullet)$) differs from the deterministic steady state because of the contribution of future shocks on their asymptotic mean. Optimal parameters for richer specifications of the tax rule are reported in Appendix C.4.

Climate policies have also an impact on the transmission of shocks to the economy. In presence of an environmental tax, any increase in output raises the value of abatement, i.e. the value of output that can be produced without changing the taxation. Equation (5.5c) indeed shows how the fraction of abated emissions, μ, positively depends on the total brown production, X_t^b. At the aggregate level this mechanism changes the effect of expansionary shocks on emissions. Consider a positive total factor productivity (TFP) shock in the euro area (Figure 10a). With no emissions taxation, the shock leads to an increase in emissions, as brown firms can boost their production without internalizing the costs. When emissions are taxed, on the other hand, any rise in output also increases the value of μ. Therefore, brown firms increment emissions abatement proportionally to the increase in their production levels. In this context a positive

Figure 8: Payoffs of the non cooperative game

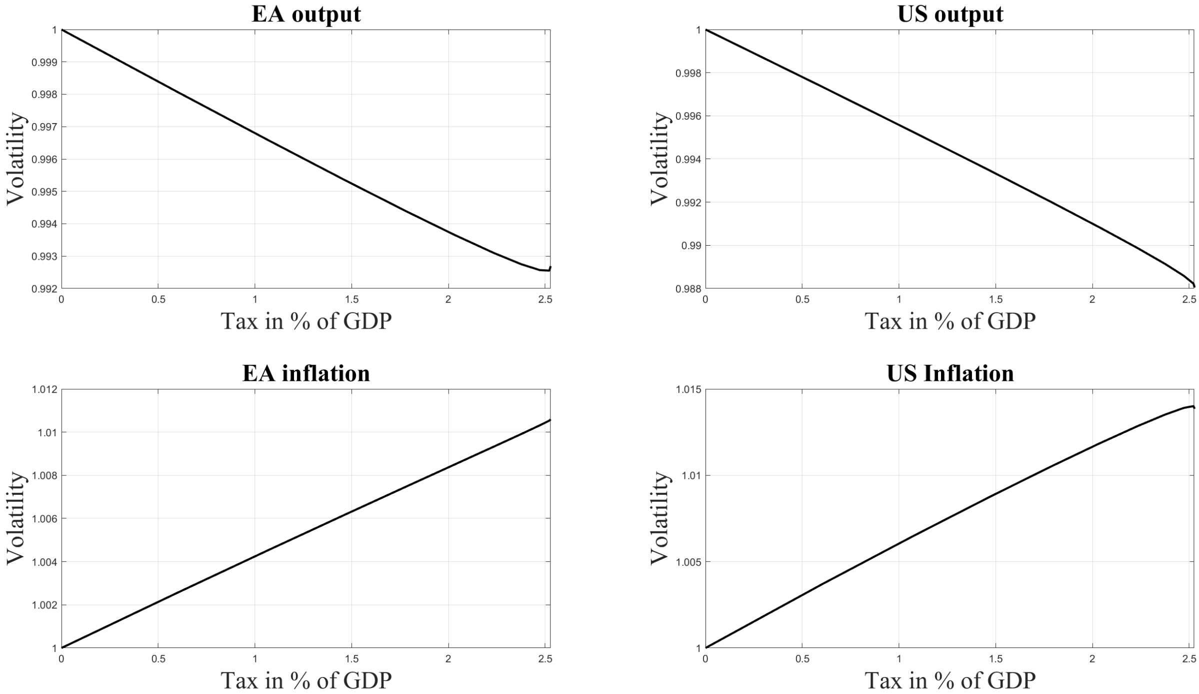


Notes: Payoffs are computed for each country as the welfare given by the combination of the policies implemented by the US (τ_{US}) and the euro area (τ_{EA}), with tax revenues transferred to households. The Nash equilibrium of the game is given by the intersection between the optimal response functions of the two countries. Welfare is defined as the unconditional mean of the recursive representation $W_t = U_t + \beta E_t(W_{t+1})$ under a second-order accurate simulation. In the chart we report total welfare. See Appendix C.5 for more details on the game and a plot of the best response functions.

TFP shock implies a *drop* in total emissions, with more evident spillovers to the foreign economy as well (Figure 10b) precisely because brown firms increase the abatement share, i.e. become "greener".

Given the welfare costs that taxing emissions implies under all the different policy setups, one might wonder what role monetary policy can play to alleviate the social costs of the green transition. Table 6 indeed reports results by assuming no reaction on the part of monetary policy. We then replicate the exercise by computing the optimal monetary policy coefficients corresponding to each optimal taxation policy. Results show that, in presence of the environmental tax, the central bank should respond relatively more, compared to the baseline, to output and relatively less to inflation (Table 7). This change in policy is sufficient to significantly reduce the welfare losses of the environmental tax. The coordination of actions between fiscal and monetary policy could then be critical to align public and private incentives towards the achievement of environmental goals. In other terms, central bank policies should look through climate events, while standing ready to support the economy to reduce the welfare costs of green transition. The changes in the central bank's policy function are in line with how the tax modifies the dynamics of the economy. When the environmental tax is present, in fact, brown prices are directly affected by the level of emissions through brown firms' marginal costs. At the same time, the taxation of emissions implies some degree of redistribution between green and brown firms and

Figure 9: Standard deviation of output and inflation for different level of the environmental tax



Notes: Second-order consistent standard deviation of output and inflation in the euro area and the US for different levels of the environmental tax. Standard deviations are standardized to 1 in the model with no taxation in both countries.

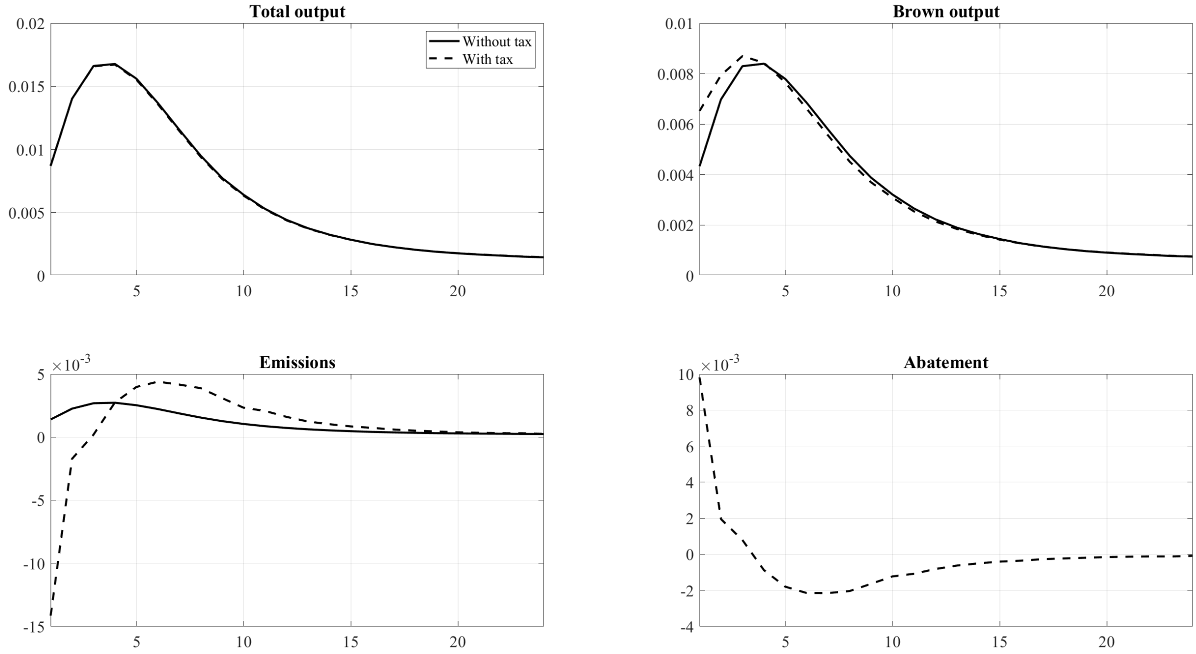
stimulates the adoption of abatement technologies within the brown sector. These dynamics, in turn, change the way inflation and output react to shock. Figure 9 plots the volatility of output and inflation for different levels of the environmental tax in both countries. Simulations show that as the environmental tax increases, the volatility of output is reduced while inflation volatility increases. In such an environment, it becomes optimal for the central bank to change its reaction function to adapt to the different economic environment.

This outcome is best achieved when both countries undertake actions to reduce emissions. According to our model, indeed, even when monetary policy supports fiscal policy, only the euro area would independently implement climate policies as that would generate a welfare gain. The US, instead, would not intervene without any commitment from the other side of the Atlantic. Given the structure of the US economy, unilateral policies generate welfare losses even when supported by the monetary authority. In a scenario without cooperation, only the euro area would reduce emissions but that would fall short of the Paris objective, by reaching only a 22% reduction in emissions at the global level.

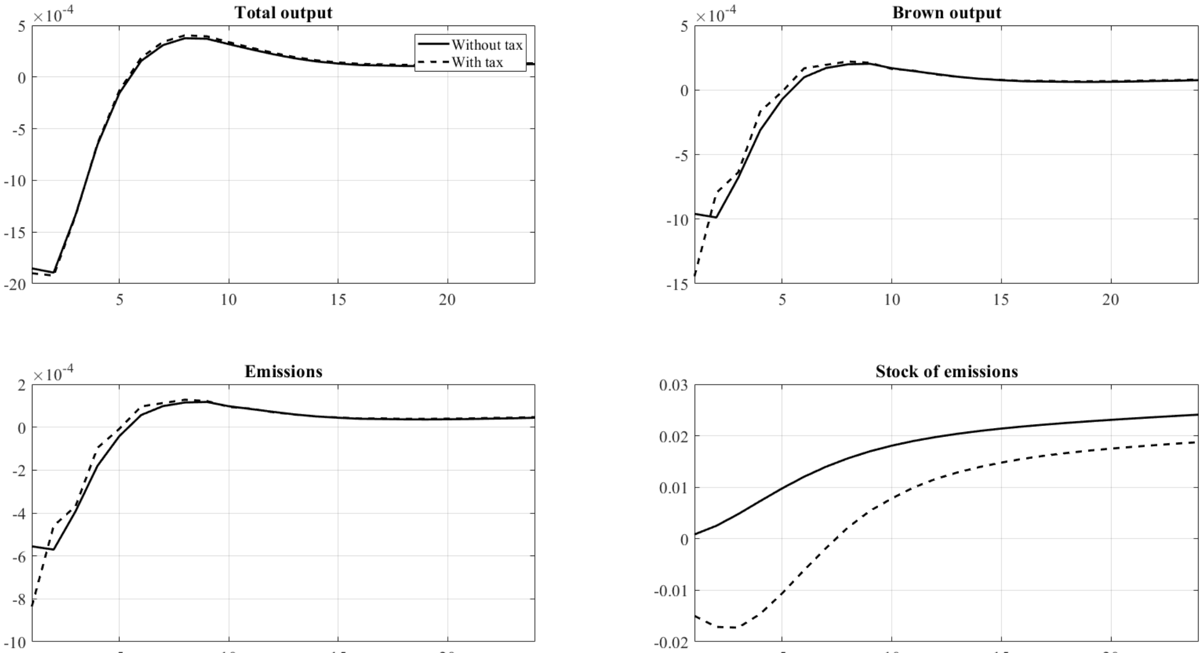
Finally, in the non-cooperative game scenario, countries aim at shifting the cost of green transition to their neighbours. Given this, the resulting Nash equilibrium entails a very limited taxation on emissions, which leads to an insufficient reduction of the environmental externality.

Figure 10: IRFs to a positive TFP shock in the euro area

(a) Euro Area



(b) US



Notes: Impulse responses to a one standard deviation positive domestic TFP shock. The black solid lines show the responses in the baseline model, without environmental taxation. The black dashed line depicts the responses with optimal taxation. Responses are expressed in deviation from the steady state and computed with a second order approximation and pruning. Stock of emissions refers to the global amount of emissions.

Monetary policy support in this case cannot significantly change the outcome.

Table 7: Monetary-fiscal policies interaction

	Revenues transferred to households				Revenues transferred to green firms			
	EA	US	Nash eq.	Coop.	EA	US	Nash eq.	Coop.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
γ_{EA}	0.60	-	0.40	0.50	0.60	-	0.44	0.50
$\theta_{\pi,EA}$	2.50	-	3.01	2.50	2.50	-	3.03	2.50
$\theta_{y,EA}$	0.00	-	0.00	0.00	0.00	-	0.00	0.00
γ_{US}	-	0.70	0.55	0.70	-	0.70	0.55	0.70
$\theta_{\pi,US}$	-	2.50	3.50	2.50	-	2.50	3.50	2.50
$\theta_{y,US}$	-	0.00	0.00	0.40	-	0.00	0.00	0.40
$E(W_{EA})$	0.30	0.10	1.15	0.30	-0.10	0.10	-0.20	-0.10
$E(W_{US})$	0.40	-0.40	-1.12	0.20	0.40	-0.70	-1.43	-0.30
$E(W_{Global})$	0.40	-0.20	0.18	0.20	0.20	-0.30	-0.83	-0.20
$E(e_{EA})$	-50.00	-0.10	-6.00	-49.80	-50.00	-0.10	-7.55	-46.00
$E(e_{US})$	-0.10	-50.00	-0.10	-50.10	0.00	-50.00	0.02	-53.30
$E(e_{Global})$	-22.60	-27.50	-2.70	-50.00	-22.60	-27.40	-3.40	-50.00

Notes: optimal monetary policy parameters under the taxation policies described in Table 6. Welfare is defined in recursive form as $W_t = U_t + \beta E_t(W_{t+1})$ and global welfare is the weighted average of welfare in the two countries: $W_{Global,t} = 0.5W_{EA,t} + 0.5W_{US,t}$. Welfare and emissions (e_{EA}, e_{US}, e_{Global}) are expressed in percent deviation from the estimated model. The second order stochastic steady state of variables ($E(\bullet)$) differs from the deterministic steady state because of the contribution of future shocks on their asymptotic mean.

5.3 Import tariffs

We define the optimal import tariff policy as the level of the import tariff which minimizes the emissions from the country targeted by the tariff[43]. We further assume that each country operates independently in setting the tariff rate, that there is no retaliation and that governments target the level of emissions of the foreign country.

Table 8 reports the change in emissions and welfare relative to the baseline model, as well as revenues from the tariff. In general, tariffs are not effective in tackling climate change as they generate relatively large welfare losses but impact emissions only marginally (in the order of 0.2% to 1.7%). Notably, the country imposing the biggest tariffs, the US, is also experiencing the largest welfare losses. There are several reasons why tariffs are so ineffective in this model. First, tariffs generate an exchange rate appreciation, thus improving the competitiveness of the taxed goods. This in turn dampens the overall impact of trade restrictions. Second, as in standard trade models, imports hit by tariffs are promptly replaced with domestic production, increasing domestic emissions and dampening the effect on the global stock of emissions x. Finally, contrary to domestic taxes, tariffs do not force firms to internalize the cost of emissions. Therefore, they do not provide incentives to abate CO_2 emissions and, hence, they do not sys-

[43] As in Lindé and Pescatori (2019), tariffs take the form of an *unexpected* and permanent increase in τ^{Imp}, after which we compute the new steady state of the economy. To identify the optimal taxation rate, we implement a grid search from 0 to 5% steady state taxation on imports.

temically change the production technologies.

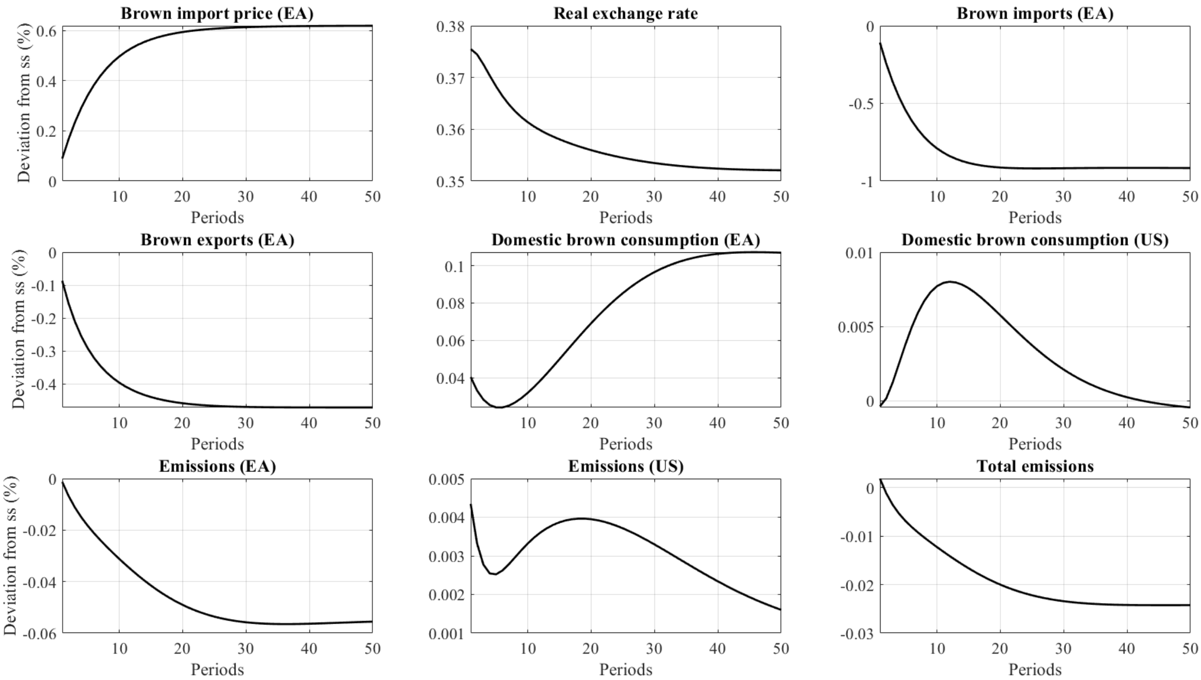
These channels are particularly strong in the euro area. When brown import tariffs are implemented, the price of brown imports in the euro area increases, whereas the imported volume decreases (see the upper panels of Figure 11) and euro area consumers buy more domestically produced brown goods. However, the tariff also induces a real exchange rate appreciation, which dampens the overall effectiveness of the policy in reducing brown imports. As the exchange rate appreciates, euro area exports, both brown and green, become less competitive. The US in turn import less brown goods from the euro area and substitute the missing imports with domestic brown products. This is highly inefficient from an aggregate prospective, because euro area brown production is less polluting than the US one. Finally, in regard to emissions, in the euro area the net effect of the tariff generates a contraction of aggregate emissions, because the increase in the consumption of domestic brown goods does not compensate for the fall in brown exports. On the contrary, in the US the net effect of the tariff is positive, albeit very small, because domestic production significantly compensates for the emissions reduction induced by lower exports to the euro area. At the aggregate level total emissions falls, but only marginally. Results are similar for the US, the only notable difference being the exchange rate reaction (Figure E.10). When the US impose the import tariff, which is four times bigger than the euro area tax (Table 8), import prices increase, thus reducing imports of brown goods from the euro area. US consumers then substitute imported goods with domestic ones, which consequently makes the consumption of domestically produced brown goods increase. Therefore, on the one hand emissions contract in the euro area, where both brown exports and domestic brown production decrease, but on the other hand they increase in the US because of the rise in domestic brown production. The drop in euro area emissions more than compensates for the increment in US emissions, which then makes global emissions fall. However, given the link between prices and emissions, the implementation of an import tariff in the US leads to a depreciation of the real exchange rate, which is commensurate to the emissions losses, $\mathcal{L}(x)$. In the US the steady state level of $\mathcal{L}(x)$ is three times bigger than in the euro area, and this generates completely different price movements across the two economies. To clarify this aspect, we compare price reaction to the same tariff across the symmetric calibration and the estimated model (Figure E.11). With the symmetric calibration, a tariff raises US prices mainly through the reaction of green goods pricing. When the cost of emission is large, as in the estimated model, the implementation of the import tariff makes green products cheaper, because the reduction in the GDP environmental cost ($\mathcal{L}(x)$) dominates the price effect of higher demand for green goods. This mechanism can get to the point of reducing US inflation, which in turn leads to an exchange rate depreciation.

Table 8: Optimal tariffs

	EA	US
$E(W_{EA})$	-0.03	-0.02
$E(W_{US})$	-0.09	-4.80
$E(W_{Global})$	-0.06	-2.46
$E(e_{EA})$	-0.06	-1.50
$E(e_{US})$	0.01	-0.05
$E(e_{Global})$	-0.02	-0.70
Tariff revenues (% GDP)	0.49	2.20

Notes: optimal tariff policy computed as in Lindé and Pescatori (2019). Welfare is defined in recursive form as $W_t = U_t + \beta E_t(W_{t+1})$ and global welfare is the weighted average of welfare in the two countries: $W_{Global,t} = 0.5 W_{EA,t} + 0.5 W_{US,t}$. Welfare and emissions (e_{EA}, e_{US}, e_{Global}) are expressed in percent deviation from the estimated model. The second order stochastic steady state of variables ($E(\bullet)$) differs from the deterministic steady state because of the contribution of future shocks on their asymptotic mean.

Figure 11: Effects of euro area tariff on brown imports from the US



Notes: Reaction to the permanent introduction of the optimal tariff in the euro area on US brown imports. Variables are expressed in percent deviations from the steady state.

5.4 Taking stock of climate policies

The outcome of the policy experiments proposed in this section seems to suggest that the imposition of a domestic tax on brown production is the most effective way to achieve a meaningful reduction in emissions. Such a direct tax is the only policy instrument in the menu that can drastically shift the incentives of firms and force some of them to change production technology, i.e. abate emissions. The emission tax can indeed move the economy to another steady state featuring a lower level of emissions both domestically and abroad. As the stock of emissions decays at a very slow rate, estimated at around 90 years (Heutel (2012)), it is necessary to change the equilibrium allocation between brown and green firms in order to have a sizeable effect on emissions and, hence, on the associated costs. The tax accomplishes this objective by increasing the production cost of brown goods and, at the same time, "greening" a share of brown production through the abatement technology. Trade barriers, instead, have a limited power in reducing emissions, because the exchange rate counteracts the price effect of tariffs and domestic polluting production substitutes imports, thus leaving global emissions essentially unchanged.

Among the possible policy alternatives taken into consideration, the use of monetary policy instruments alone appears to be the least effective. This result is not surprising as monetary policy is meant to stabilize the economy around the business cycle, rather than affecting the forces that determine long-term growth. Given this, in the class of models which our framework belongs to different policy rules are irrelevant for the deterministic steady state, in that they aim at smoothing the volatility of the economy. However, the volatility of variable matters when using higher-order solutions, thus affecting the stochastic steady state of the model. In this case, then, monetary policy can have an impact on welfare and other endogenous variables *at higher order*, as long as volatility has a relevant impact on those variables[44]. That in turn depends on the behavioural equations of agents and on the policy function. In the model, emissions are not particularly affected by uncertainty therefore they do not react significantly to changes in volatility while they are mostly determined, also at higher orders, by their deterministic steady state level. Because monetary policy operates through volatility, changes in the monetary policy rule have little effects in reducing emissions. The same results holds when looking at unconventional monetary policies such as in Ferrari and Nispi Landi (2020).

Despite this, monetary policy plays a crucial role in making the green transition possible. As shown in Table 6, none of the mitigating policies is incentive-compatible (i.e. it increases the welfare of the implementing country). As a result, the two countries do not have the incentive to implement any of them. There are many reasons why climate policies might reduce welfare. First, the introduction of the tax distorts the equilibrium allocation of resources, which entails both lower profits for firms and lower financial returns to households. Second, the tax has effects

[44]An example of this mechanism is provided by welfare. As households' utility functions are concave, i.e. households are risk-adverse, the volatility of the economy significantly affects the value of welfare and, because of that, different policy rules can impact the second-order stochastic steady state.

on inflation, because it increases the cost of brown goods, thus reducing real returns and wages. Third, as countries are not symmetric, the introduction of a tax might have disproportionate effects on the terms of trade and, hence, divert trade flows. Reducing emissions has also positive effects through the decrease in GDP losses, $\mathcal{L}(x)$, but that is insufficient to overcome the costs for regions like the euro area where the cost of climate events is still relatively small. A key result of our paper is that monetary policy plays a crucial role in enabling the climate transition. By adapting the policy framework to the new, greener structure of the economy, central banks can make the transition possible by partially shielding households from the costs of the transition. As reported in Table 7 this entails a lower reaction to inflation and a somewhat stronger reaction to output relative to the optimal policy in the absence of the environmental tax, Table 4. These changes are the outcome of the shift in the overall structure of the economy which is induced by the implementation of the tax. In such a context, central banks need to adapt their reaction function to the new environment in order to fulfill their mandate. Another key element to succeed in tackling climate change is international cooperation. Only when both countries commit to reduce emissions, the global climate objective can be reached with welfare gains on both sides.

Additional insights on the effectiveness of the different policy configurations are provided by a the inspection of the transition dynamics from the old to the new (greener) equilibrium. While the transition for the main macroeconomic aggregates is relatively fast (Figures 13 and E.12), on the other hand the reduction in the stock of emissions, which determines the severity of climatic adverse events and, hence, the materialisation of losses, is much slower (Figure 12)[45]. In this context, the policy setting whereby countries cooperate to enact a combination of fiscal and monetary containment measures entails the biggest reduction (-17%) in the stock of emissions over a 50-year horizon.

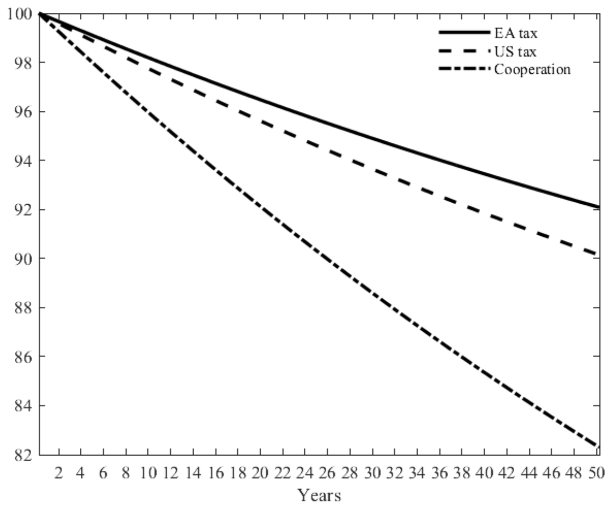
6 Conclusion

In this paper we set up a model that, to the best of our knowledge, is the first to investigate the international implications of climate mitigation policies and fiscal-monetary interactions in a DSGE framework. Our work is aimed at filling a gap in the literature and nesting the debate on climate change into an international macro setting.

Specifically, we first document cross-country empirical regularities in the relationship between emissions and output. We then rationalize those findings through the lens of a structural esti-

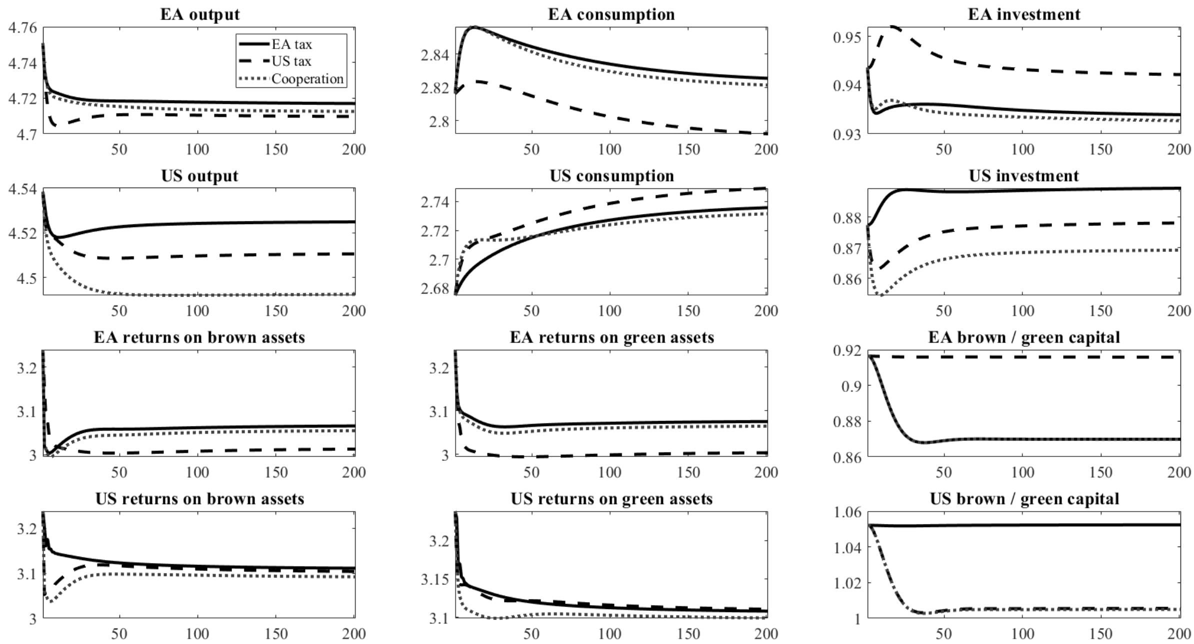
[45]The persistence in the stock of emissions is due to the high value of the parameter ρ^x in the model. Following Heutel (2012), ρ^x is calibrated to be 0.9979 in both countries (half-life of around 90 years).

Figure 12: Transition of global emissions stock for different policy configurations



Notes: transition of emissions stock between the equilibrium without containment policy to the *equilibria* reported in Columns (1), (2) and (4) of Table 7. Emission stock is normalised to 100 when the policy is implemented.

Figure 13: Transition of endogenous variables between *equilibria*



Notes: transition between the equilibrium without containment policy to the *equilibria* reported in Columns (1), (2) and (4) of Table 7. Variables are expressed in levels.

mated macro model for the US and the euro area. We show how emissions have generally fallen after the global financial crisis, but they have somewhat taken up in more recent years in the US, with concerning spillovers to the euro area.

We then use the estimated model to evaluate three possible policies that can be deployed to attain a reduction in emissions which is compatible with the Paris agreement target: i) monetary policy; ii) domestic emissions taxation; iii) tariffs on brown imports.

Our results suggest that the most effective policy is a combination of domestic taxes on emissions and a change in the monetary policy reaction function, so to accommodate the economic transition towards a greener economy and adapt to changes in volatility of output and inflation when containment policies are active. Notably, international cooperation, i.e. the commitment to target the aggregate level of emissions on the part of both countries, is equally crucial to achieve the Pareto-improving outcome of lowering emissions without welfare losses, which is the only one incentive-compatible sustainable equilibrium. In particular, countries that are bearing the highest costs from climate changes have also the highest incentives to put mitigating policies in place, because the ensuing benefits would be larger. That equilibrium, however, can be achieved only through coordinated actions. If countries act independently and non-cooperatively, individual incentives would prevent them from implementing an appropriate level of climate-mitigating policies to meet the emissions reduction target. Under the non-cooperative equilibrium, indeed, the environmental tax is too low to achieve the climate objective, while still entailing moderate welfare losses. Brown import tariffs, finally, are ineffective in reducing emissions because their direct effect on the import of brown goods is dampened by exchange rate movements. Moreover, the implementation of an import tariff also induces a substitution of brown imported goods with domestic production, thus reducing the effect on the global level of emissions.

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Appendix

A Data

Table A.1 reports the list of variables used both in Section 2 and in the estimation of the model in Section 4, together with their sources and the time coverage.

Table A.1: Data sources

Variable	Source		Time Coverage
	Euro area	US	
Empirical analysis			
CO_2 emissions[1]	OECD[¶]	OECD	2000Q1-2018Q4
Real GDP	SDW[+]	FRED[*]	2000Q1-2018Q4
Real GDP per capita	OECD	FRED	2000Q1-2018Q4
Inflation	SDW	FRED	2000Q1-2018Q4
Return on Assets	Bloomberg	Bloomberg	2000Q1-2018Q4
Shadow rate[2]	Atlanta FED[§]	Atlanta FED	2000Q1-2018Q4
Population	OECD	FRED	2000Q1-2018Q4
Model estimation			
Working age population (15-65)	OECD	US-BLS[†]	2000Q1-2018Q4
Consumption	Eurostat	FRED	2000Q1-2018Q4

Notes : [1] Annual data have been linearly interpolated to obtain quarterly series;
[2] for euro area, MRO rate is used before 2004Q3.
Sources: [¶] OECD data [+] ECB Statistical Data Warehouse; [*] FED of St. Louis FRED;
[§] Wu and Xia's shadow rates; [†] US Bureau of Labor Statistics

B Derivations

B.1 Households

Households' optimization problem is:

$$\max U_t = E_t \sum_{j=0}^{\infty} \beta^j \left[e_t^c \ln \left(C_{t+j} - hC_{t+j-1} \right) - \frac{\chi}{1+\phi} L_{t+j}^{1+\phi} \right], \tag{B.1}$$

subject to

$$\begin{aligned} P_t C_t + B_t^H + NER_t B_t^F + \sum_{i=b,g} P_t I_t^i \leq \sum_{i=b,g} W_t^i L_t^i + R_t B_{t-1}^H + \\ + R_t^* NER_t B_{t-1}^F - \frac{\phi^B}{2} \left(\frac{NER_t B_t^F}{P_t} \right)^2 P_t + \\ + \sum_{i=b,g} P_t R_t^{k,i} K_{t-1}^i + \Pi_t \end{aligned} \tag{B.2}$$

and

$$K_{t+1}^i = \left\{ (1-\delta^i) K_t^i + I_t^i \left[1 - \frac{\phi^K}{2} \left(\frac{I_t^i}{I_{t-1}^i} - 1 \right)^2 \right] \right\} \tag{B.3}$$

where Equation (B.1) is the household's budget constraint, Equation (B.3) is the law of motion of capital in each production sector, e_t is a consumption preference shock and δ^i is the depreciation rate of capital in sector $i \in \{b, g\}$. Equation (B.3) also includes adjustment costs for the installation of new capital.

In addition, aggregate labor supply is:

$$L_t = \left[\chi^b (L_t^b)^{1+\sigma_L} + \chi^g (L_t^g)^{1+\sigma_L} \right]^{\frac{1}{1+\sigma_L}} \tag{B.4}$$

The optimization problem entails the following first order conditions:

$$\frac{e_t^c}{C_t - hC_{t-1}} - E_t\frac{\beta e_{t+1}^c h}{C_{t+1} - hC_t} = \Lambda_t \tag{B.5a}$$

$$\chi^b \left[(L_t)^{\phi+\sigma_L}\right](L_t^b)^{\sigma_L} = \Lambda_t \frac{W_t^b}{P_t} \tag{B.5b}$$

$$\chi^g \left[(L_t)^{\phi+\sigma_L}\right](L_t^g)^{\sigma_L} = \Lambda_t \frac{W_t^g}{P_t} \tag{B.5c}$$

$$\beta E_t\left(\Lambda_{t+1}\frac{R_t}{\pi_{t+1}}\right) = \Lambda_t \tag{B.5d}$$

$$\Lambda_t\left[1 + \phi^B\left(\frac{NER_t B_t^F}{P_t}\right)\right] = \beta E_t\left(\Lambda_{t+1}\frac{R_t^*}{\pi_{t+1}}\frac{NER_{t+1}}{NER_t}\right) \tag{B.5e}$$

$$Q_t^i\left\{\left[1 - \frac{\phi^K}{2}\left(\frac{I_t^i}{I_{t-1}^i} - 1\right)^2\right] - \frac{I_t^i}{I_{t-1}^i}\phi^K\left(\frac{I_t^i}{I_{t-1}^i} - 1\right)\right\} +$$

$$+\beta E_t\left[Q_{t+1}^i\phi^K\left(\frac{I_t^i}{I_{t-1}^i} - 1\right)\left(\frac{I_{t+1}^i}{I_t^i}\right)^2\right] = \Lambda_t \tag{B.5f}$$

$$\beta E_t\left[Q_{t+1}^i(1-\delta^i) + \Lambda_{t+1}R_{t+1}^{k,i}\right] = Q_t^i \tag{B.5g}$$

where $i \in \{b, g\}$, $\left[1 + \phi^B\left(\frac{NER_t B_t^F}{P_t}\right)\right]$ are cross-country bond holding costs and Equation (B.5f) is the Tobin's Q equation for each asset class. First order conditions are exactly symmetric for the foreign economy.

B.2 Production

In each country there are two *continua* of perfectly competitive firms, indexed by k and j and producing with the brown and green technology respectively. Brown firms' production function is:

$$X_t^b(k) = [1 - \mathcal{L}(x_t)]A_t\left(K_t^b(k)\right)^\alpha\left(L_t^b(k)\right)^{1-\alpha}, \tag{B.6}$$

where x_t is a function of the aggregate production of brown firms in the domestic and foreign economies. As producers are atomistic in nature, they choose the optimal level of production by taking x_t as given. Moreover, in Equation (B.6) $\mathcal{L}(x_t) = d_1 x_t^{d2}$ and $X_t^b(k) = X_{dom,t}^b(k) + X_{exp,t}^b(k)$. Cost minimization implies:

$$R_t^{k,b}(k) = [1 - \mathcal{L}(x_t)]A_t MC_t^b(k)\alpha\left(K_t^b(k)\right)^{\alpha-1}\left(L_t^b(k)\right)^{1-\alpha} \tag{B.7a}$$

$$\frac{W_t^b(k)}{P_t} = [1 - \mathcal{L}(x_t)]A_t MC_t^b(k)(1-\alpha)\left(K_t^b(k)\right)^\alpha\left(L_t^b(k)\right)^{-\alpha} \tag{B.7b}$$

where MC_t^b is the Lagrangian multiplier associated to the optimization problem of firms and also represents the real marginal cost of production.

The production function of green firms is:

$$X_t^g(j) = [1 - \mathcal{L}(x_t)] A_t \left(K_t^g(j)\right)^{\alpha\psi} \left(L_t^g(j)\right)^{1-\alpha} \tag{B.8}$$

with total output being the sum of goods sold domestically and abroad $X_t^g(j) = X_{dom,t}^g(j) + X_{exp,t}^g(j)$. Cost minimization implies:

$$R_t^{k,g}(j) = [1 - \mathcal{L}(x_t)] A_t MC_t^g(j) \alpha\psi \left(K_t^g(j)\right)^{\alpha\psi} \left(L_t^g(j)\right)^{1-\alpha} \tag{B.9a}$$

$$\frac{W_t^g(j)}{P_t} = [1 - \mathcal{L}(x_t)] A_t MC_t^g(j)(1-\alpha) \left(K_t^g(j)\right)^{\alpha\psi} \left(L_t^g(j)\right)^{-\alpha} \tag{B.9b}$$

where MC_t^g is the Lagrangian multiplier associated to the optimization problem of green firms. The marginal productivity of capital differs across brown and green firms if $\psi \neq 1$. In addition, green firms suffer from the same productivity loss due to emissions as brown firms, even though they do not contribute to the stock of emissions.

B.3 Retailers

In both home and foreign economies, each sector produces perfectly competitive intermediate goods. Y_{exp}^i and Y_{dom}^i denote export and domestic demand, $i \in \{H, F\}$. We adopt the following demand aggregators:

$$\begin{aligned} C_{H,t}^b + I_{H,t}^b + G_{H,t}^b &= Y_{dom,t}^b, \quad C_{H,t}^g + I_{H,t}^g + G_{H,t}^g = Y_{dom,t}^g \\ C_{F,t}^b + I_{F,t}^b + G_{F,t}^b &= Y_{exp,t}^b, \quad C_{F,t}^g + I_{F,t}^g + G_{F,t}^g = Y_{exp,t}^g. \end{aligned} \tag{B.10}$$

$\forall i \in \{b, g\}$, $Y_{dom,t}^i$ and $Y_{exp,t}^b$ are composites of goods purchased by perfectly competitive firms from monopolists producing with the technology:

$$\begin{aligned} Y_{dom,t}^b &= \left[\int_0^1 X_{dom,t}^b(k)^{\frac{\nu-1}{\nu}} dk\right]^{\frac{\nu}{\nu-1}}, \quad Y_{dom,t}^g = \left[\int_0^1 X_{dom,t}^g(j)^{\frac{\nu-1}{\nu}} dj\right]^{\frac{\nu}{\nu-1}} \\ Y_{exp,t}^b &= \left[\int_0^1 X_{exp,t}^b(k)^{\frac{\nu-1}{\nu}} dk\right]^{\frac{\nu}{\nu-1}}, \quad Y_{exp,t}^g = \left[\int_0^1 X_{exp,t}^g(j)^{\frac{\nu-1}{\nu}} dj\right]^{\frac{\nu}{\nu-1}}, \end{aligned} \tag{B.11}$$

where ν is the elasticity of substitution across different goods. We denote the indexing of firms in the brown and green sector by k and j respectively.

Price aggregators are:

$$
\begin{aligned}
P^b_{dom,t} &= \left[\int_0^1 P^b_{dom,t}(k)^{1-\nu}dk\right]^{\frac{1}{1-\nu}}, \quad P^g_{dom,t} &= \left[\int_0^1 P^g_{dom,t}(j)^{1-\nu}dj\right]^{\frac{1}{1-\nu}} \\
P^b_{exp,t} &= \left[\int_0^1 P^b_{exp,t}(k)^{1-\nu}dk\right]^{\frac{1}{1-\nu}}, \quad P^g_{exp,t} &= \left[\int_0^1 P^g_{exp,t}(j)^{1-\nu}dj\right]^{\frac{1}{1-\nu}}
\end{aligned}
\tag{B.12}
$$

with $P^i_{dom,t}$ and $P^b_{exp,t}$ begin the prices of domestically consumed and exported goods. Demand functions are:

$$
\begin{aligned}
X^b_{dom,t}(k) &= \left[\frac{P^b_{dom,t}(k)}{P^b_{dom,t}}\right]^{-\nu} Y^b_{dom,t}, \quad X^g_{dom,t}(j) &= \left[\frac{P^g_{dom,t}(j)}{P^g_{dom,t}}\right]^{-\nu} Y^g_{dom,t} \\
X^b_{exp,t}(k) &= \left[\frac{P^b_{exp,t}(k)}{P^b_{exp,t}}\right]^{-\nu} Y^b_{exp,t}, \quad X^g_{exp,t}(j) &= \left[\frac{P^g_{exp,t}(j)}{P^g_{exp,t}}\right]^{-\nu} Y^g_{exp,t}.
\end{aligned}
\tag{B.13}
$$

Final consumption goods are created by combining brown goods from country H and F ($C^b_{H,t}$, $C^b_{F,t}$) and green goods from country H and F ($C^g_{H,t}$, $C^g_{F,t}$). Aggregate consumption, C_t, then is:

$$
\begin{aligned}
C_t = \Big\{&[\omega\Upsilon]^{1-\varrho}\left(C^b_{H,t}\right)^{\varrho} + [\omega\,(1-\Upsilon)]^{1-\varrho}\left(C^g_{H,t}\right)^{\varrho} + \\
&+ [(1-\omega)\,\Upsilon]^{1-\varrho}\left(C^b_{F,t}\right)^{\varrho} + [(1-\omega)\,(1-\Upsilon)]^{1-\varrho}\left(C^g_{F,t}\right)^{\varrho}\Big\}^{\frac{1}{\varrho}}
\end{aligned}
\tag{B.14}
$$

where ω is the share of home goods in consumption and Υ is the share of brown goods in consumption. Similarly, total government consumption, G_t is:

$$
\begin{aligned}
G_t = \Big\{&[\omega\Upsilon]^{1-\varrho}\left(G^b_{H,t}\right)^{\varrho} + [\omega\,(1-\Upsilon)]^{1-\varrho}\left(G^g_{H,t}\right)^{\varrho} + \\
&+ [(1-\omega)\,\Upsilon]^{1-\varrho}\left(G^b_{F,t}\right)^{\varrho} + [(1-\omega)\,(1-\Upsilon)]^{1-\varrho}\left(G^g_{F,t}\right)^{\varrho}\Big\}^{\frac{1}{\varrho}}
\end{aligned}
\tag{B.15}
$$

where $G^i_{H,t}$ is government consumption of domestic goods and $G^i_{F,t}$ is government consumption of foreign goods, with $i \in \{b, g\}$. Retailers' profits are:

$$
\Pi^C_t = P_t\,(C_t)^{\frac{1}{\varrho}} - \sum_{i=g,b}\left(P^i_{H,t}C^i_{H,t} + P^i_{F,t}C^i_{F,t}\right) \tag{B.16a}
$$

$$
\Pi^G_t = P_t\,(G_t)^{\frac{1}{\varrho}} - \sum_{i=g,b}\left(P^i_{H,t}G^i_{H,t} + P^i_{F,t}G^i_{F,t}\right) \tag{B.16b}
$$

Optimality conditions define the demand functions:

$$
\begin{aligned}
C^b_{H,t} &= \left(\frac{P^b_{H,t}}{P_t}\right)^{\frac{1}{\varrho-1}} \omega\Upsilon C_t, & C^g_{H,t} &= \left(\frac{P^g_{H,t}}{P_t}\right)^{\frac{1}{\varrho-1}} \omega(1-\Upsilon)C_t \\
C^b_{F,t} &= \left(\frac{P^b_{F,t}}{P_t}\right)^{\frac{1}{\varrho-1}} (1-\omega)\Upsilon C_t, & C^g_{F,t} &= \left(\frac{P^g_{F,t}}{P_t}\right)^{\frac{1}{\varrho-1}} (1-\omega)(1-\Upsilon)C_t
\end{aligned}
\tag{B.17}
$$

$$
\begin{aligned}
G^b_{H,t} &= \left(\frac{P^b_{H,t}}{P_t}\right)^{\frac{1}{\varrho-1}} \omega\Upsilon G_t, & G^g_{H,t} &= \left(\frac{P^g_{H,t}}{P_t}\right)^{\frac{1}{\varrho-1}} \omega(1-\Upsilon)G_t \\
G^b_{F,t} &= \left(\frac{P^b_{F,t}}{P_t}\right)^{\frac{1}{\varrho-1}} (1-\omega)\Upsilon G_t, & G^g_{F,t} &= \left(\frac{P^g_{F,t}}{P_t}\right)^{\frac{1}{\varrho-1}} (1-\omega)(1-\Upsilon)G_t.
\end{aligned}
\tag{B.18}
$$

The aggregate price level is then obtained by substituting Equation (B.17) into Equation (B.16):

$$
P_t = \left[\omega\Upsilon\left(P^b_{H,t}\right)^{\frac{\varrho}{\varrho-1}} + \omega(1-\Upsilon)\left(P^g_{H,t}\right)^{\frac{\varrho}{\varrho-1}} + (1-\omega)\Upsilon\left(P^b_{F,t}\right)^{\frac{\varrho}{\varrho-1}} + (1-\omega)(1-\Upsilon)\left(P^g_{F,t}\right)^{\frac{\varrho}{\varrho-1}}\right]^{\frac{\varrho-1}{\varrho}}.
\tag{B.19}
$$

As the shares ω and Υ are the same across goods, aggregate prices are also the same.

B.4 Monopolists

Each type of good $i \in \{b, g\}$ is sold on the final markets by monopolists who set prices with some degree of market power. In each period, monopolists can update prices with probability ξ. Otherwise, prices remain fixed to the previous period's level. A monopolist selling brown goods in the home country and who can update her price chooses $\overline{P^b_{H,t}(k)}$ and $\overline{P^b_{F,t}(k)}$ to maximise her profits:

$$
\begin{aligned}
E_t \sum_{j=0}^{\infty} \beta^j \Lambda_{t+j} \Bigg\{ & \left[\frac{\overline{P^b_{H,t}(k)}}{P_{H,t+j}}(1+\tau_b) - MC^b_{t+j}\right]\left(\frac{\overline{P^b_{H,t}(k)}}{P_{H,t+j}}\right)^{-\nu} Y^b_{H,t} + \\
& + \left[\frac{NER_{t+j}\overline{P^b_{F,t}(k)}}{P_{F,t+j}}(1+\tau_b) - MC^b_{t+j}\right]\left(\frac{\overline{P^b_{F,t}(k)}}{P_{F,t+j}}\right)^{-\nu} Y^b_{F,t}\Bigg\},
\end{aligned}
\tag{B.20}
$$

where MC^i_t is the marginal cost of producing good i and τ_i a production tax on good i, with $i \in \{b, g\}$. Furthermore, $\Lambda_t \equiv E_t\left(\frac{\lambda_{t+1}}{\lambda_t}\right)$ is the stochastic discount factor. The optimality condition for domestic profits then is:

$$
E_t \sum_{j=0}^{\infty} (\xi\beta)^j \Lambda_{t+j} \left[\frac{\overline{P^b_{H,t}(k)}}{P_t}\frac{P_t}{P_{t+j}}\left(\frac{P^b_{H,t}}{P^b_{H,t+j}}\right)^{-\nu} Y^b_{H,t+j} - \frac{1}{1+\tau_b}\frac{\nu}{\nu-1} MC^b_{t+j}\left(\frac{P^b_{H,t}}{P^b_{H,t+j}}\right)^{-\nu} Y^b_{H,t+j}\right] = 0
\tag{B.21}
$$

Equation (B.21) can be written as[46]:

$$\mathcal{F}^b_{H,t}\frac{\overline{P^b_{H,t}}}{P_t} = \mathcal{K}^b_{H,t} \tag{B.22}$$

with:

$$\mathcal{F}^b_{H,t} = \sum_{j=0}^{\infty}(\xi\beta)^j \Lambda_{t+j}\frac{P_t}{P_{t+j}}\left(\frac{P^b_{H,t}}{P^b_{H,t+j}}\right)^{-\nu} Y^b_{H,t+j}$$
$$\mathcal{K}^b_{H,t} = \sum_{j=0}^{\infty}(\xi\beta)^j \Lambda_{t+j}\frac{1}{1+\tau_b}\frac{\nu}{\nu-1}MC^b_{t+j}\left(\frac{P^b_{H,t}}{P^b_{H,t+j}}\right)^{-\nu} Y^b_{H,t+j}$$

The optimal price for exports ($\overline{P^b_{F,t}(k)}$) solves:

$$\begin{aligned} E_t\sum_{j=0}^{\infty}(\xi\beta)^j \Lambda_{t+j}\Bigg[\frac{NER_{t+j}}{NER_t}\frac{NER_tP^*_t}{P_t}\frac{\overline{P^b_{F,t}(k)}}{P^*_t}\frac{P_t}{P_{t+j}}\left(\frac{P^b_{F,t}}{P^b_{F,t+j}}\right)^{-\nu} Y^b_{F,t+j}+ \\ -\frac{1}{1+\tau_b}\frac{\nu}{\nu-1}MC^b_{t+j}\left(\frac{P^b_{F,t}}{P^b_{F,t+j}}\right)^{-\nu} Y^b_{F,t+j} = 0 \end{aligned} \tag{B.23}$$

Equation (B.23) can be rewritten as:

$$\mathcal{F}^b_{F,t}RER_t\frac{\overline{P^b_{F,t}(k)}}{P^*_t} = \mathcal{K}^b_{F,t} \tag{B.24}$$

with

$$\mathcal{F}^b_{F,t} = \sum_{j=0}^{\infty}(\xi\beta)^j \Lambda_{t+j}\frac{NER_{t+j}}{NER_t}\frac{P_t}{P_{t+j}}\left(\frac{P^b_{F,t}}{P^b_{F,t+j}}\right)^{-\nu} Y^b_{F,t+j}$$
$$\mathcal{K}^b_{F,t} = \sum_{j=0}^{\infty}(\xi\beta)^j \Lambda_{t+j}\frac{1}{1+\tau_b}\frac{\nu}{\nu-1}MC^b_{t+j}\left(\frac{P^b_{F,t}}{P^b_{F,t+j}}\right)^{-\nu} Y^b_{F,t+j}.$$

The CPI price index is:

$$\begin{aligned} P^b_{H,t} &= \left[(1-\xi)(\overline{P^b_{H,t}})^{1-\nu} + \xi(P^b_{H,t-1})^{1-\nu}\right]^{\frac{1}{1-\nu}} \\ P^b_{F,t} &= \left[(1-\xi)(\overline{P^b_{F,t}})^{1-\nu} + \xi(P^b_{F,t-1})^{1-\nu}\right]^{\frac{1}{1-\nu}}, \end{aligned} \tag{B.25}$$

while inflation rates are $\pi^b_{H,t} = \frac{P^b_{H,t}}{P^b_{H,t-1}}$ and $\pi^b_{F,t} = \frac{P^b_{F,t}}{P^b_{F,t-1}}$. Optimality conditions are symmetric both for green goods and in the foreign economy.

[46] As all firms choose the same optimal price when possible, we drop the index k for notation convenience.

B.5 Capital producers

Capital producers provide investment goods to brown and green firms by using domestic and imported final goods. Aggregate investments are:

$$
\begin{aligned}
I^b &= \left[\omega^{1-\rho}\left(I^b_{H,t}\right)^{\rho} + (1-\omega)^{1-\rho}\left(I^b_{F,t}\right)^{\rho}\right]^{\frac{1}{\rho}} \\
I^g &= \left[\omega^{1-\rho}\left(I^g_{H,t}\right)^{\rho} + (1-\omega)^{1-\rho}\left(I^g_{F,t}\right)^{\rho}\right]^{\frac{1}{\rho}}.
\end{aligned}
\tag{B.26}
$$

Profits are:

$$
\begin{aligned}
\Pi^b_t &= I^b_t - P^b_{H,t} I^b_{H,t} - P^b_{F,t} I^b_{F,t} \\
\Pi^g_t &= I^g_t - P^g_{H,t} I^b_{H,t} - P^g_{F,t} I^g_{F,t}.
\end{aligned}
\tag{B.27}
$$

Demand schedules for investment goods are:

$$
\begin{aligned}
I^i_{H,t} &= \left(\frac{P^i_{H,t}}{P_t}\right)^{\frac{1}{\rho-1}} I^i_t \\
I^i_{F,t} &= \left(\frac{P^i_{F,t}}{P_t}\right)^{\frac{1}{\rho-1}} I^i_t
\end{aligned}
\tag{B.28}
$$

with $i \in \{b, g\}$.

B.6 Public sector

Government consumption is exogenous and evolves as follows:

$$
\ln\left(\frac{G_t}{G_{ss}}\right) = \rho_g \ln\left(\frac{G_{t-1}}{G_{ss}}\right) + \varepsilon^G_t. \tag{B.29}
$$

On the other hand, monetary policy is characterized by a Taylor-type rule, which depends on inflation and output growth:

$$
\ln R_t = \gamma_r \ln R_{t-1} + (1-\gamma_r)\left[\ln R_{ss} + \theta_\pi \ln \pi_t + \theta_Y \left(\ln Y_t - \ln Y_{t-1}\right)\right] + \varepsilon^r_t. \tag{B.30}
$$

B.7 Aggregation

In each country aggregate supply equals aggregate demand:

$$
Y_t = C_t + I_t + G_t \tag{B.31}
$$

with total output Y_t being equal to the sum of outputs of the brown and green sectors: $Y_t = Y_t^b + Y_t^g$. Total production in each industry is:

$$\begin{aligned} Y_t^b &= \int_0^1 \left(\frac{P_{dom,t}^b(k)}{P_t}\right)^{-\nu} Y_{dom,t}^b dk + \int_0^1 \left(\frac{P_{exp,t}^b(k)}{P_t}\right)^{-\nu} Y_{exp,t}^b dk \\ Y_t^g &= \int_0^1 \left(\frac{P_{dom,t}^g(j)}{P_t}\right)^{-\nu} Y_{dom,t}^g dj + \int_0^1 \left(\frac{P_{exp,t}^g(j)}{P_t}\right)^{-\nu} Y_{exp,t}^g dj. \end{aligned} \tag{B.32}$$

Equation (B.32) can be also solved in recursive form for $i \in \{b, g\}$ as:

$$d_{dom,t}^i Y_{dom,t}^i + d_{exp,t}^i Y_{exp,t}^i = Y_t^i, \tag{B.33}$$

with the dispersion terms defined as:

$$\begin{aligned} d_{dom,t}^i &= (1-\xi)(p_{dom,t}^i)^{\nu}(\overline{p_{dom,t}^i})^{-\nu} + \xi(\pi_{dom,t}^i)^{\nu} d_{dom,t-1}^i \\ d_{exp,t}^i &= (1-\xi)(p_{exp,t}^*)^{\nu}(\overline{p_{exp,t}^i})^{-\nu} + \xi(\pi_{exp,t}^i)^{\nu} d_{exp,t-1}^i, \end{aligned} \tag{B.34}$$

where $p_{dom,t}^i$ and $p_{exp,t}^i$ are the real price of domestic and exported goods, $\overline{p_{dom,t}^i}$ and $\overline{p_{exp,t}^i}$ the real *optimal* prices of domestic and exported goods, $\pi_{dom,t}$ and $\pi_{exp,t}$ the corresponding inflation rates.

The stock of emissions, which is the same in the two economies, evolves according to:

$$x_t = \rho^x x_{t-1} + e_t + e_t^* \tag{B.35}$$

where

$$\begin{aligned} e_t &= \mathcal{A}_t (X_t^b)^{1-\gamma} \qquad \text{and} \\ e_t^* &= \mathcal{A}_t^* (X_t^{*,b})^{1-\gamma} \end{aligned} \tag{B.36}$$

are the new emissions. Finally, bond markets are cleared in each period, with zero net supply:

$$\begin{aligned} B_t^H + B_t^{*,F} &= 0 \\ B_t^{*,H} + B_t^F &= 0. \end{aligned} \tag{B.37}$$

C Extensions

C.1 The role of emissions in model predictions

In order to get a sense of how the main mechanics of the model change in presence of emissions, we first consider a calibrated version where countries are perfectly symmetric.

Figure C.1 compares the deterministic steady state of endogenous variables relative to the model without the brown sector externality[47]. The negative effect of higher emissions on steady state variables grows quasi-linearly in the associated output loss. Notably, when $\mathcal{L}(x_{ss}) = 6\%$ output, consumption, capital stock and investments in the steady state are around 9% lower than in an emission-free environment.

As to the dynamic response of endogenous variables to shocks, Figure C.2 plots the reaction of domestic and foreign aggregates to a 1% brown sector TFP shock in the domestic economy for different levels of emissions. We find that emission externalities reduce the expansionary effects of a TFP shock in the economy, with output increasing on impact by 6% and 16% less in the scenarios with medium and high emissions respectively. This leads to likewise reductions in the positive effects on demand, exports and investments in the home economy, as well as on demand, exports and investments in the foreign country. The policy rate and inflation, on the other hand, have pretty similar reactions for different levels of emissions[48]. This finding is in line with the empirical evidences presented in Section 2 and suggests a limited effect of CO_2 emissions on the price level.

C.2 Optimal monetary policy in the face of emission losses

In this section we examine the optimal monetary policy problem of the domestic central bank which seeks to define the optimal response to output and inflation in order to maximise households welfare. The households' welfare function is:

$$Welf_t = U_t + \beta E_t Welf_{t+1} \tag{C.1}$$

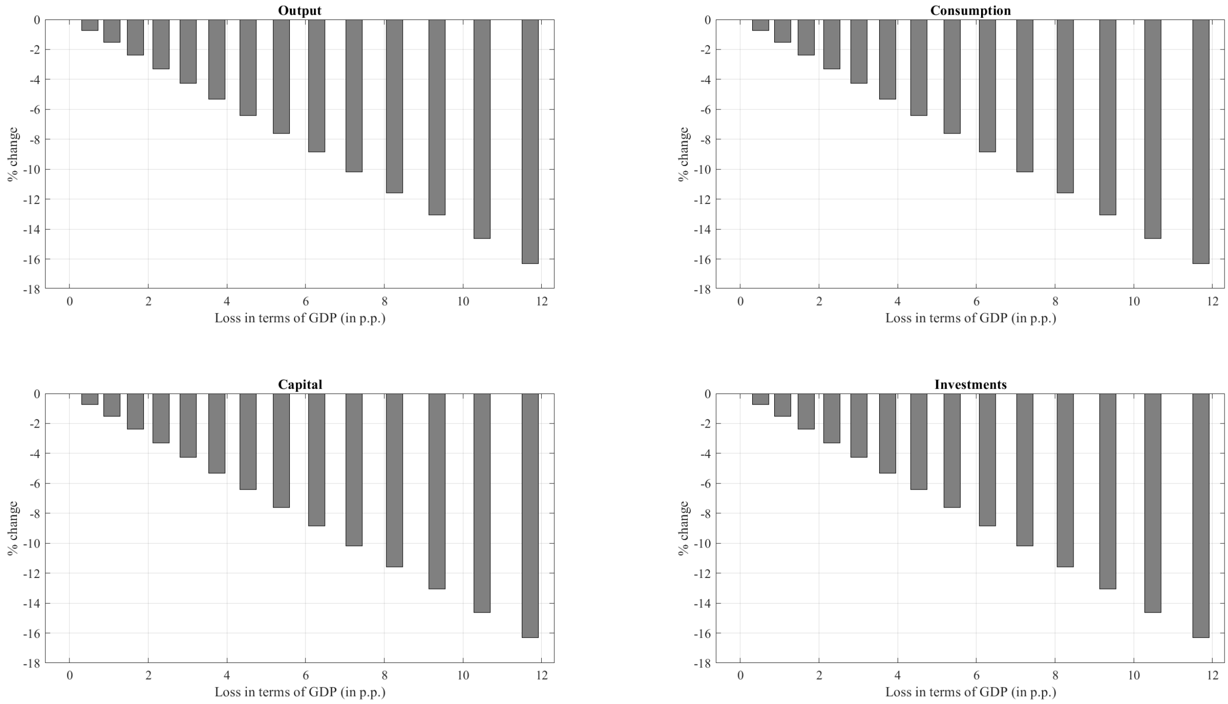
the central bank set the policy rate according to a Taylor-time rule:

$$\ln R_t = \gamma_r \ln R_{t-1} + (1 - \gamma_r) \left[\ln R_{ss} + \theta_\pi \ln \pi_t + \theta_Y \left(\ln Y_t - \ln Y_{t-1}\right)\right] + \varepsilon_t^r \tag{C.2}$$

[47] See Table D.6 for the complete list of calibrated parameters.

[48] Figure E.8 in the Appendix reports the cumulative IRFs of a larger set of endogenous variables and levels of emissions for a monetary policy shock, both in home (Figure E.8a) and foreign (Figure E.8b) economies.

Figure C.1: Changes in the steady state of endogenous variables (in percentage points) relative to the model without the externality in production of the brown sector



Notes: Each bar corresponds to a different calibration of the loss function $\mathcal{L}(\cdot)$ that delivers a percent loss of steady state output reported on the x-axis.

hence the central bank optimally chooses γ_r, θ_π and θ_Y to maximise Equation (C.1) subject to all the other equations in the model. Results for different levels of the steady state loss are reported in Table C.2.

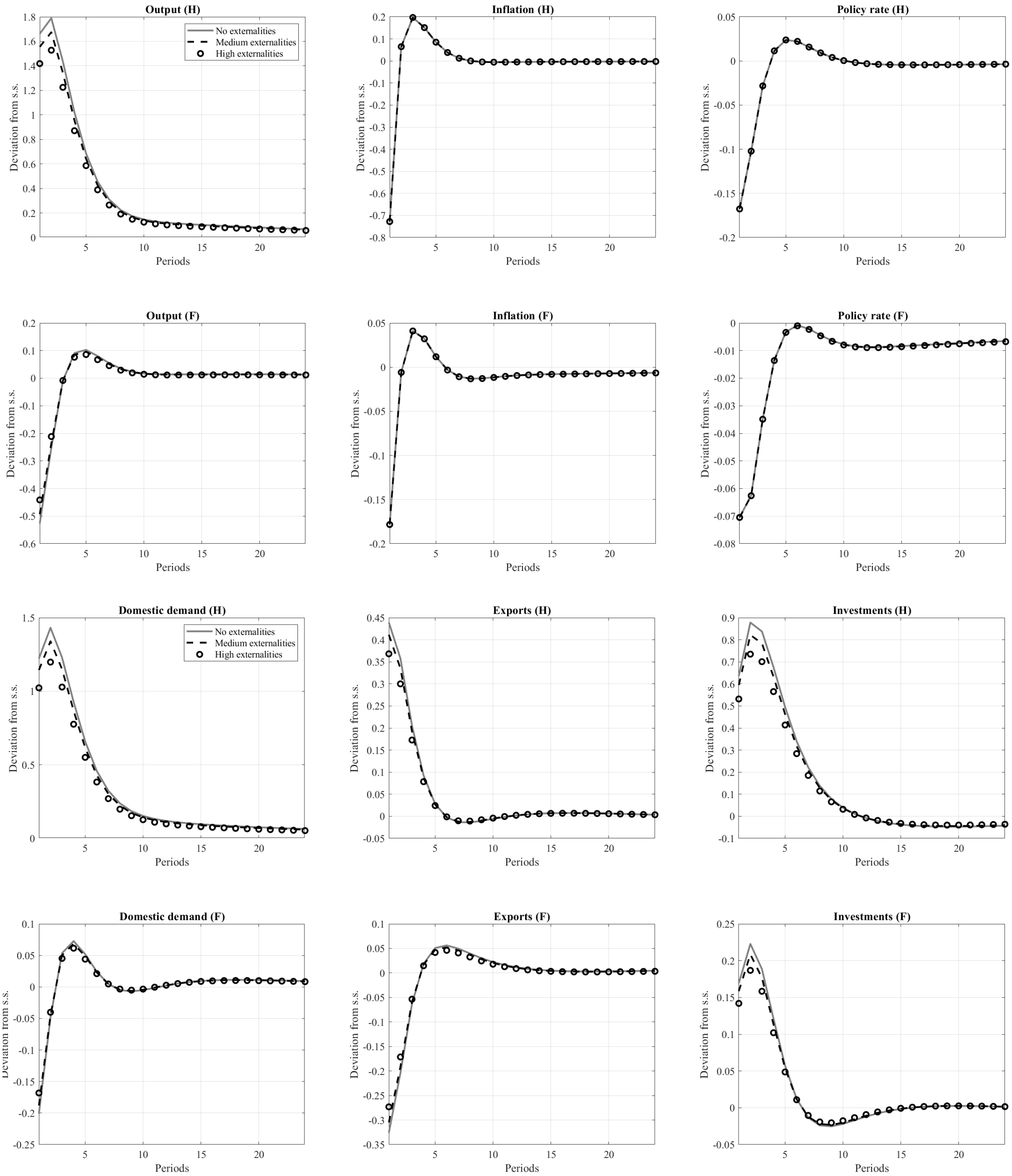
In an environment without emissions, it is optimal for the central bank to stabilize inflation, a standard result in the Neo-Keynesian literature. The same applies when emission externalities are low. Above a certain threshold for the externalities, however, the central bank has to start considering output as well in its policy response.

C.3 Targeting emissions instead of welfare for monetary policy

A natural question would be to examine an alternative scenario where the central bank targets also the level of emissions. Specifically, we assume that the central bank optimizes a weighted average of welfare deviations from its optimal level and the distance of emissions from the level compatible with the Paris 2 agreement, that's to say a reduction of emissions by up to 50% (IMF (2019)). The loss function is specified as:

$$Loss = w\left[\frac{E(W)}{\bar{W}} - 1\right]^2 + (1-w)\left[\frac{E(e)}{\bar{e}} - 1\right]^2 \tag{C.3}$$

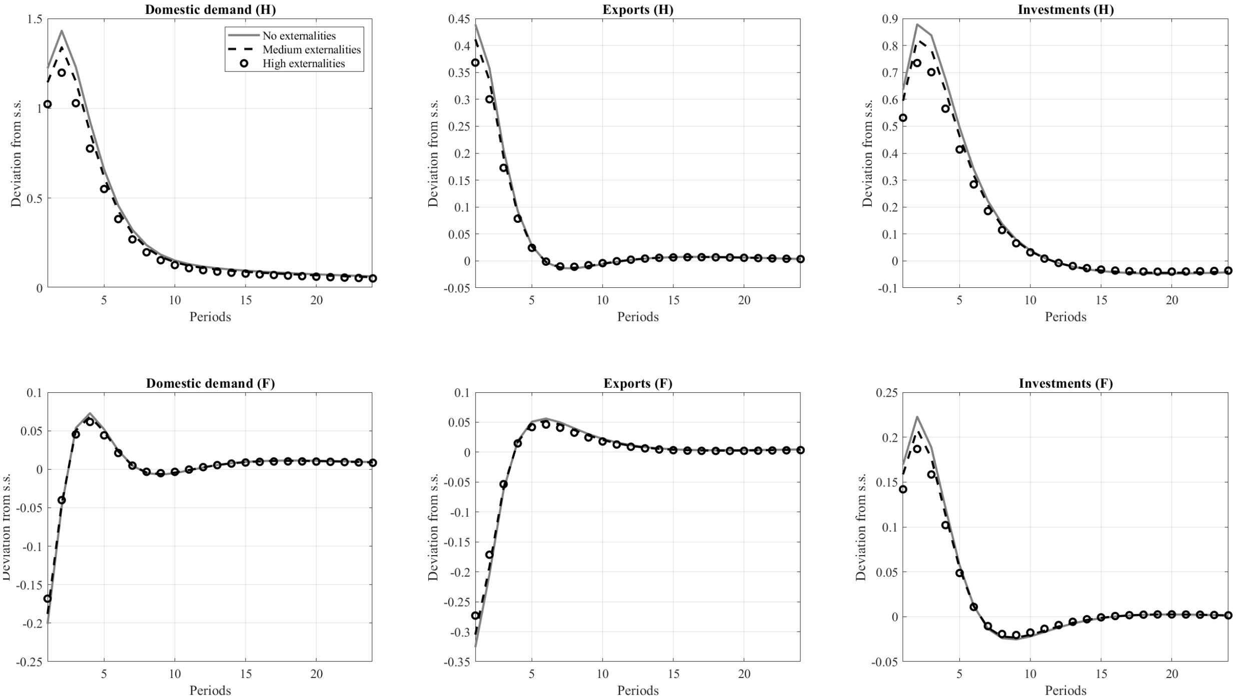
Figure C.2: Impulse response of macro variables in the domestic (H) and foreign (F) economy to a 1% positive TFP shock in the domestic brown sector



Notes: The grey solid line represents the response of the model without emissions, the black dashed line the response of the model with a medium level of emissions (implying a 4% loss of steady state output) and the black dots depict the response of the model with an high level of emissions (implying a 9% loss of steady state output). Responses are expressed in deviation from the steady state.

where $E(W)$ and (e) are welfare and emissions in the stochastic steady state, $\bar{W}$ and $\bar{e}$ are their target values and w is the weight of welfare in the objective function. The welfare target is set

Figure C.3: Impulse response of macro variables in the domestic (upper panel) and foreign (lower panel) economy to a 1% positive TFP shock in the domestic brown "sector".



Notes: The grey solid line reports the response of the model without emissions, the black dashed line the response of the model with a medium level of emissions (implying a 4% loss of steady state output) and the dashed line the response of the model with an high level of emissions (around 9% of steady state output).

Table C.2: Optimal monetary policy

	$\mathcal{L}(x) = 0$	$\mathcal{L}(x) = 2\%$	$\mathcal{L}(x) = 4\%$
γ_r	0.2	0.2	0.2
$(1-\gamma_r)\theta_\pi$	1.6	1.6	1.6
$(1-\gamma_r)\theta_Y$	0	0	0.8
θ_π	2	2	2
θ_Y	0	0	1
$std(Y_H)$	3.9502	3.8898	3.4950
$std(Y_F)$	3.7885 0	3.7311	3.3555
$std(\pi_H)$	1.9593	1.9593	1.9592
$std(\pi_F)$	1.9507	1.9507	1.9508

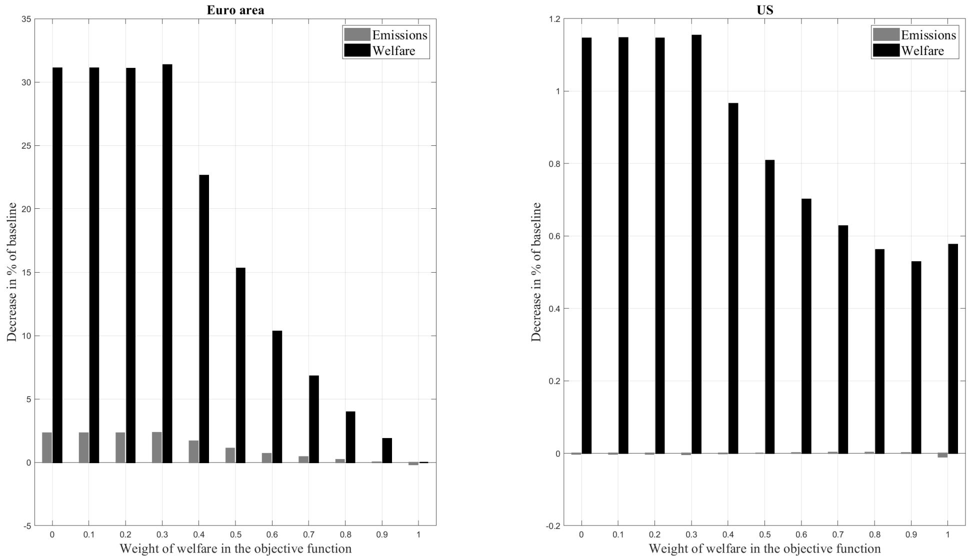
Notes: optimal choice of the parameters in the monetary policy rule of the central bank in the domestic economy for different levels of steady state emissions. Welfare is defined as the stochastic steady state of households' welfare $Welf_t = U_t + \beta E_t Welf_{t+1}$ at second order.

equal to the optimal policy welfare, while the target for emissions coincides with a reduction by 50%. Equation (C.3) has a global minimum at $E(W) = \bar{W}$ and $E(e) = \bar{e}$. The parameter w determines the relative preferences between welfare and emissions. When $w = 1$ the objective of the central bank coincides with the standard optimal policy framework discussed in Section 5.1, because the global minimum can be achieved only when welfare is at the optimal level. When, instead, $w = 0$ the optimum is reached when new emissions are 50% lower than in the baseline version in the stochastic steady state. In this latter case the impact on welfare is not considered by the monetary policy authority. Results for different values of w are reported in Figure C.4. Even under a full climate objective (i.e. $w = 0$) monetary policy has limited effects in reducing the stock of emissions, which remains far above the Paris 2 objective. Additionally, simulations suggests that reductions in emissions are associated to large welfare losses relative to the optimal policy. These results are in line with Ferrari and Nispi Landi (2020), who show that monetary policy has limited effects in reducing emissions. This is mainly due to the fact that monetary policy in macro models aims at stabilizing the business cycle (volatility) around a pre-determined trend (steady state)[49]. Tackling climate change, on the other hand, requires an altogether shift in both the structure of the economy and the incentives of private agents, i.e. the steady state, which cannot then been achieved with standard monetary policy instruments. More technically, changes in monetary policy affect the derivatives in $h_{\sigma\sigma}$ and $g_{\sigma\sigma}$ of Equation (5.1), which however have low loadings on x.

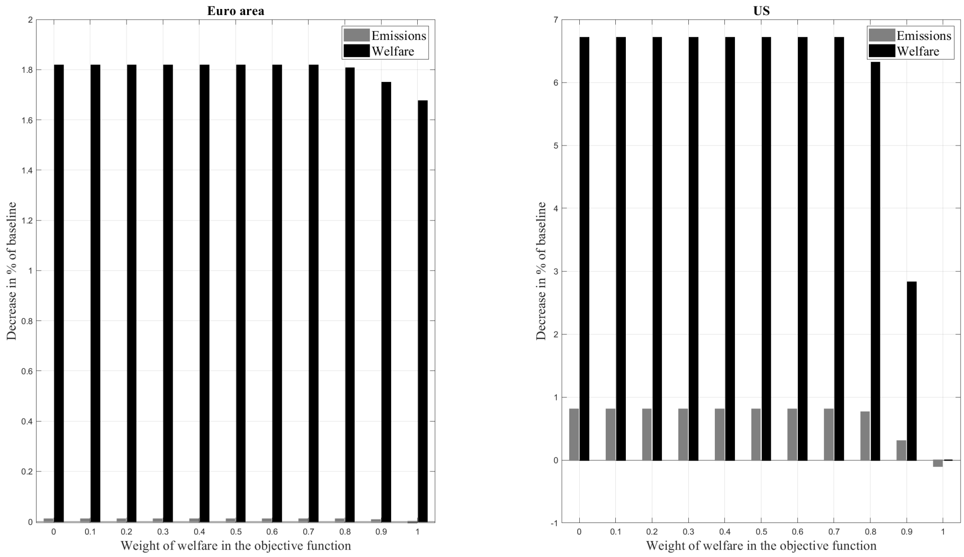
[49] See Woodford (2003).

Figure C.4: Reduction in welfare and emissions, relative to the estimated model, under optimal policy

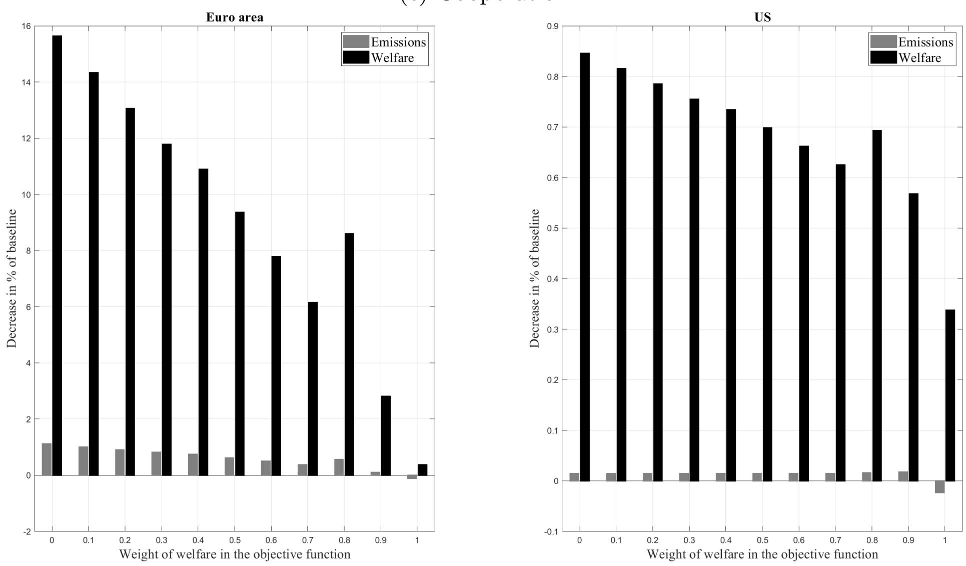
(a) Optimal policy in the euro area



(b) Optimal policy in the US



(c) Cooperation



Notes: Decrease in welfare and emissions, relative to the estimated model, for different values of w, under the optimal policy when the central banks objective is both welfare and a reduction in emissions. Welfare is defined as $W_t = U_t + \beta E_t(W_{t+1})$ while the objective of emissions is a reduction of 50%.

C.4 Alternative taxation rules

In this section, we re-estimate the model by taking into consideration alternative taxation policies, whereby the tax rate τ is defined by a steady state level τ_{ss} and a component that reacts to the business cycle. In particular, we consider a policy rule that is function of three indicators, namely output (Y_t), output by brown firms (X_t^b) and the price of brown capital (Q_t^b):

$$\tau_t = \tau_{ss} + \theta_{\tau,y}(\ln Y_t - \ln Y_{t-1}) + \theta_{\tau,b}(\ln X_t^b - \ln X_{t-1}^b) + \theta_{\tau,q}(\ln Q_t^b - \ln Q_{t-1}^b). \tag{C.4}$$

Results in Table C.3 correspond to the case where $\theta_{\tau,b}$ and $\theta_{\tau,q}$ in Equation (C.4) are both set equal to 0. Table C.4, on the other hand, reports all coefficients estimates. We do not find significant differences with the baseline model results discussed in Section 5.

Table C.3: Optimal fiscal policy with reaction to output

	Climate externalities			No climate externalities		
	EA	US	Coop.	EA	US	N Coop.
	(1)	(2)	(3)	(4)	(5)	(6)
$\tau_{ss,EA}$	0.07	0.00	0.06	0.07	0.00	0.06
$\theta_{\tau,EA}$	-0.08	0.00	-0.02	-0.08	0.00	-0.02
$\tau_{ss,US}$	0.00	0.06	0.06	0.00	0.06	0.06
$\theta_{\tau,US}$	0.00	-0.06	-0.09	0.00	-0.06	-0.09
$E(W_{EA})$	-1.16	0.08	-1.21	-1.60	0.06	-1.63
$E(W_{US})$	0.39	-0.80	-0.04	0.40	-1.11	-0.53
$E(W_{Global})$	-0.36	-0.37	-0.61	-0.57	-0.54	-1.06
$E(e_{EA})$	-50.00	-0.15	-47.17	-50.00	-0.06	-47.23
$E(e_{US})$	-0.11	-50.00	-52.33	-0.05	-50.00	-52.29
$E(e_{Global})$	-22.65	-27.48	-50.00	-22.62	-27.44	-50.00
Tax EA (% GDP)	1.15	0.00	1.11	1.15	0.00	1.11
Tax US (% GDP)	0.00	1.19	1.22	0.00	1.18	1.21

Notes: optimal coefficients of tax policy rule under the cooperative and non cooperative equilibria and in presence or absence of climate externalities in the model. Welfare is defined in recursive form as $W_t = U_t + \beta e_t(W_{t+1})$ and global welfare is the weighted average of welfare in the two countries: $W_{Global,t} = 0.5W_{EA,t} + 0.5W_{US,t}$. Welfare and emissions (e_{EA}, e_{US}, e_{Global}) are expressed in percent deviation from the estimated model. The second order stochastic steady state of variables ($E(\bullet)$) differs from the deterministic steady state because of the contribution of future shocks on their asymptotic mean.

Table C.4: Optimal fiscal policy with reaction to all macro variables

	Climate externalities			No climate externalities		
	EA	US	Coop.	EA	US	Coop.
	(1)	(2)	(3)	(4)	(5)	(6)
$\tau_{ss,EA}$	0.07	-	0.05	0.07	-	0.05
$\theta_{\tau,y,EA}$	0.01	-	-0.03	0.01	-	-0.03
$\theta_{\tau,b,EA}$	-0.04	-	0.00	-0.04	-	0.00
$\theta_{\tau,q,EA}$	0.01	-	-0.02	0.01	-	-0.02
$\tau_{ss,US}$	-	0.06	0.07	-	0.06	0.07
$\theta_{\tau,y,US}$	-	0.01	-0.02	-	0.01	-0.02
$\theta_{\tau,b,US}$	-	-0.03	0.03	-	-0.03	0.03
$\theta_{\tau,q,US}$	-	0.01	-0.01	-	0.01	-0.01
$E(W_{EA})$	-0.76	0.07	-0.67	-1.20	0.07	-1.06
$E(W_{US})$	-0.02	-0.81	-0.58	0.01	-1.11	-1.03
$E(W_{Global})$	-0.38	-0.38	-0.62	-0.57	-0.54	-1.05
$E(e_{EA})$	-50.00	-0.15	-44.30	-50.00	-0.06	-44.31
$E(e_{EA})$	-0.08	-50.00	-54.70	-0.02	-50.00	-54.69
$E(e_{Global})$	-22.63	-27.48	-50.00	-22.60	-27.44	-50.00
Tax EA (% GDP)	1.10	0.00	0.99	1.10	0.00	0.98
Tax US (% GDP)	0.00	1.15	1.23	0.00	1.14	1.22

Notes: optimal coefficients of tax policy rule under the cooperative and non cooperative equilibria and in presence or absence of climate externalities in the model. Welfare is defined in recursive form as $W_t = U_t + \beta E_t(W_{t+1})$ and global welfare is the weighted average of welfare in the two countries: $W_{Global,t} = 0.5W_{EA,t} + 0.5W_{US,t}$. Welfare and emissions (e_{EA}, e_{US}, e_{Global}) are expressed in percent deviation from the estimated model. The second order stochastic steady state of variables ($E(\bullet)$) differs from the deterministic steady state because of the contribution of future shocks on their asymptotic mean.

C.5 Non-cooperative game

In the non-cooperative game we compute the Nash equilibrium of a game where both countries set the emission tax τ to maximize domestic welfare given the structure of the economy and the choice of the opponent. Countries solve the problem:

$$\max_{\tau_i} E(W_{i,t})|(\tau_{j\neq i}, \Omega_i), \quad i, j \in \{EA, US\} \tag{C.5}$$

where Ω_i defines the set of structural equations of the model and shocks. The Nash equilibrium of the game is defined as the intersection of the optimal response functions of each country, i.e. the sequence of strategies $\{\tau_i|\tau_{j\neq i}\}$ that solves Equation (C.5) for each possible strategy of the opponent. Practically, these are computed with the following algorithm:

1. for each value of the opponent's strategy $\tau_{j\neq i}$ we compute the optimal taxation level in country i, i.e. the value of τ_i that maximises welfare. This is the optimal response of i to the policy j. We consider a grid from $\tau = 0$ to $\tau = 0.15$, which is more than twice as large as the tax rate needed to meet the climate objective;

2. we get one optimal response for each value of $\tau_{j\neq i}$ in the grid; these are collected in the vector $\vec{\tau_i^*}|j$ which describes the optimal policy response function for i;

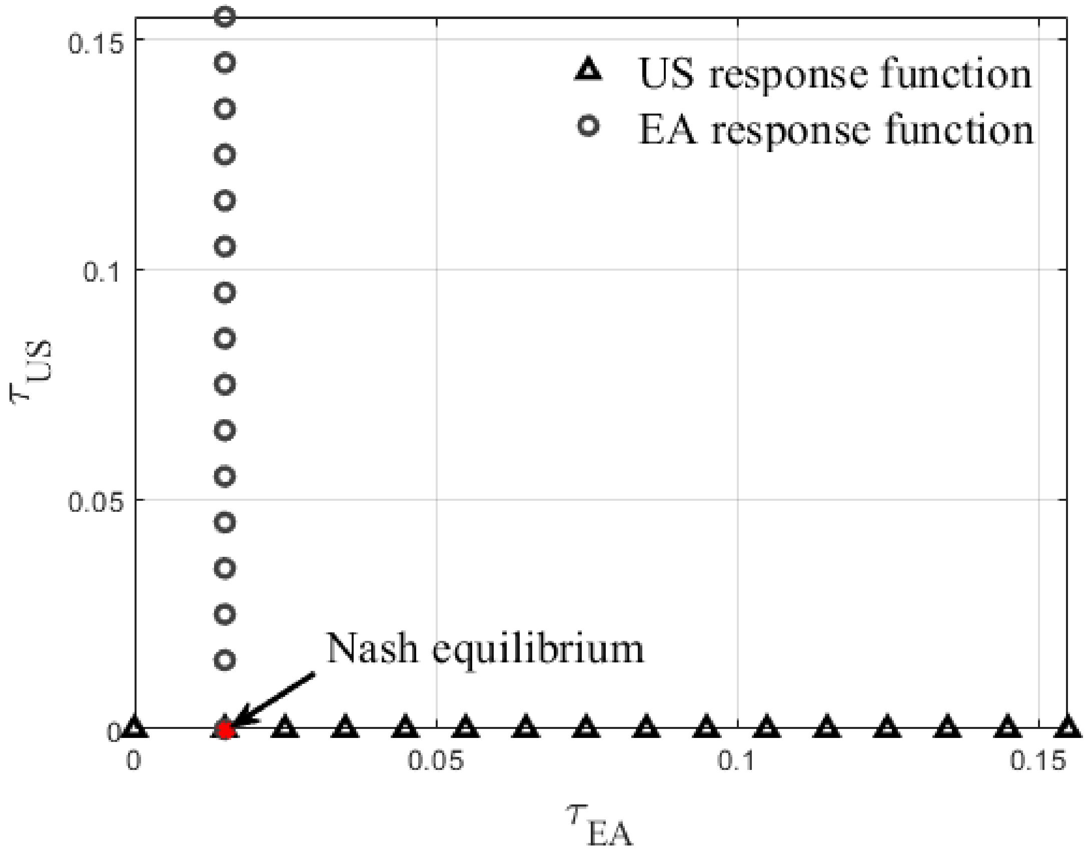
3. we apply the same process to country j to get j's optimal response function $\vec{\tau_j^*}|i$;

4. the intersections between the two best response functions are the Nash equilibria of the game

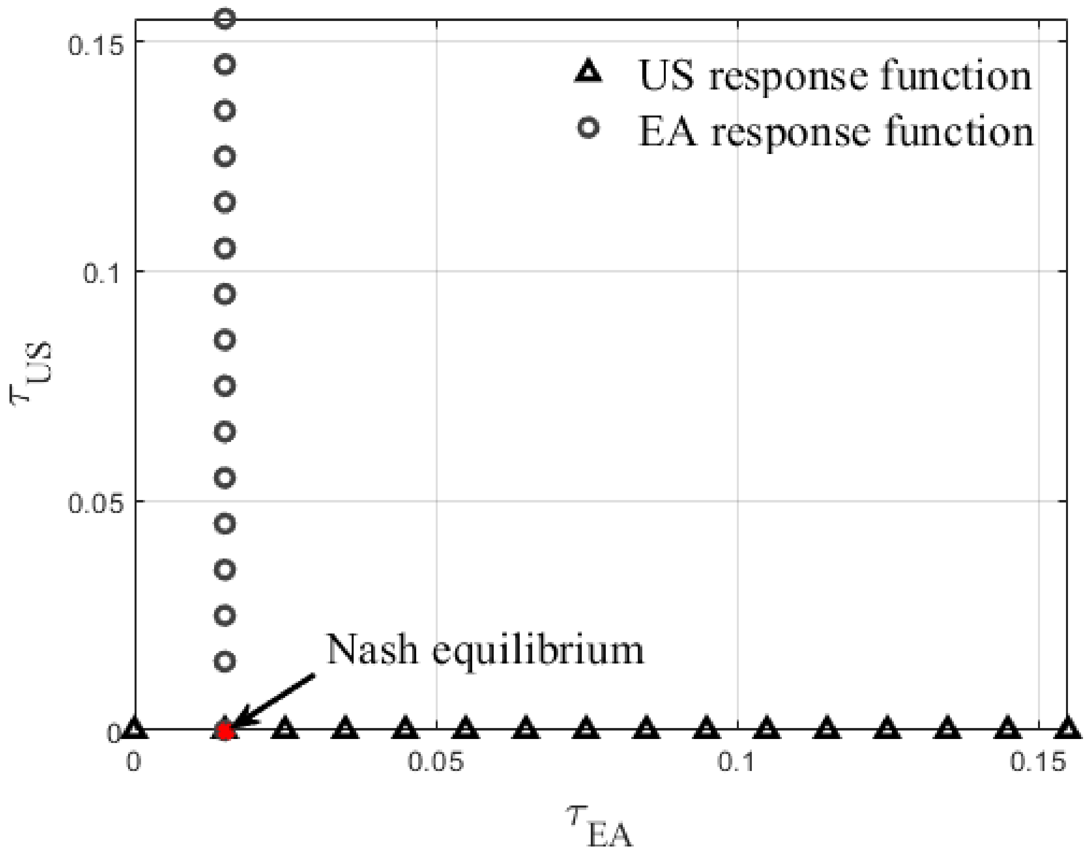
The intersection of the two reaction functions is the Nash equilibrium of the game, i.e. the strategy that maximises domestic welfare given the response of the foreign country. Each player internalizes the opponent's reaction, thus having the incentive to shift the costs of climate policies to the other country. In the resulting equilibrium, countries would refrain from implementing policies that might be globally optimal but, in combination with the opponent's reaction, deliver domestic welfare losses. The non-cooperative equilibrium, therefore, could be very far not only from the global optimal, but also from individual countries' best allocations. These dynamics are evident when displaying the reaction functions (Figure C.5). Both countries have a dominant strategy, i.e. a strategy that is optimal independently of the opponent's reaction. This strategy involves no taxation in the US and a very small tax rate in the euro area.

Figure C.5: Response functions of the non-cooperative game

(a) Tax revenues are transferred to households

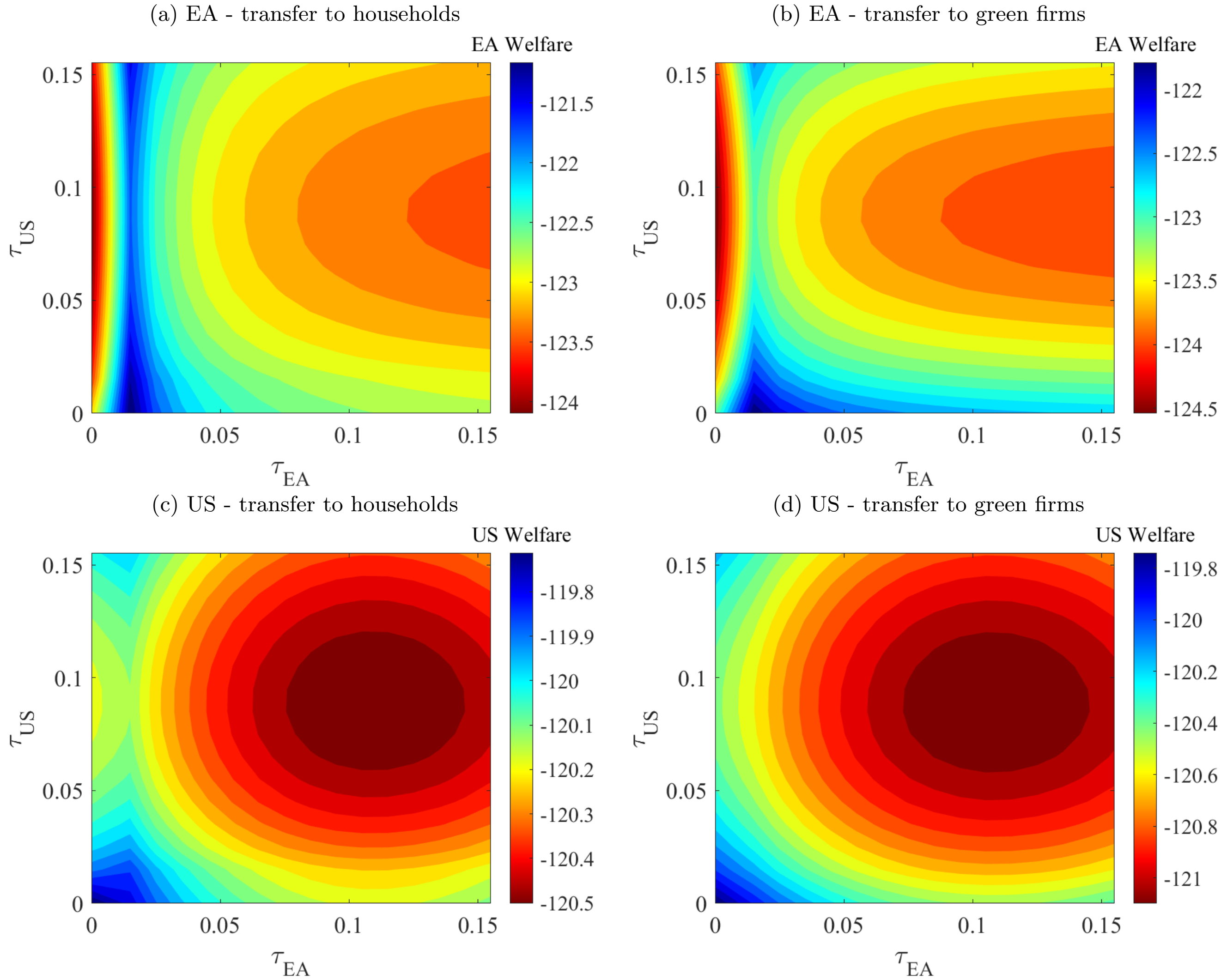


(b) Tax revenues are transferred to green firms



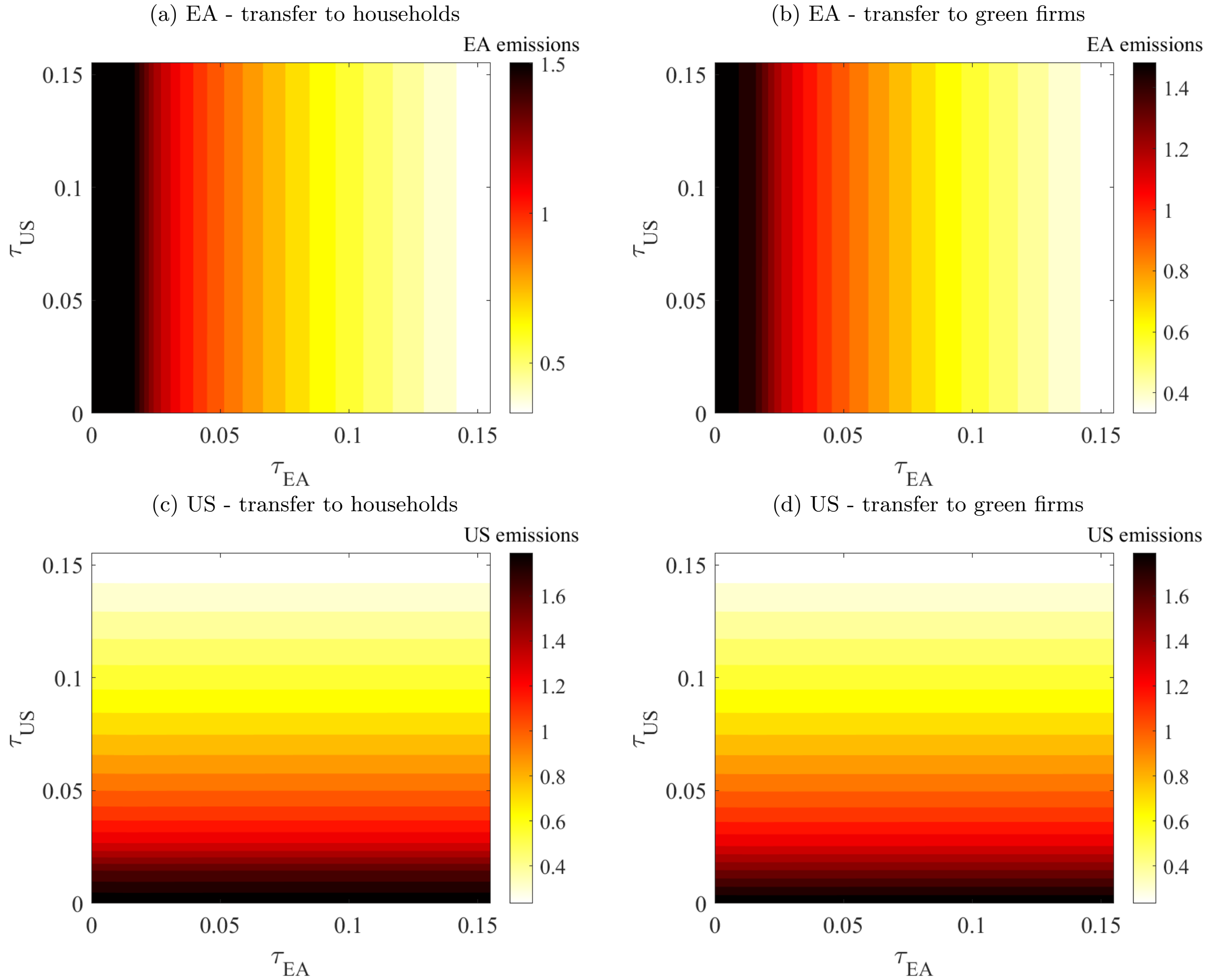
Notes: The figure plots the reaction function of both countries, i.e. the set of policies that maximises domestic welfare conditional to the reaction of the opponent. Both countries have a dominant strategy.

Figure C.6: Payoff matrices of the non-cooperative game



Notes: The figure plots the payoff matrix of the game for both countries when the tax revenues are transferred to either households or green firms. Each element of the matrix is the resulting welfare, the payoff of the game, for a specific pair of tax parameters, τ_{EA} and τ_{US}. Welfare is defined as the unconditional mean of the welfare function computed using a second-order approximation of the model.

Figure C.7: Emission matrices of the non-cooperative game



Notes: The figure plots the emission matrix of the game for both countries when the tax revenues are transferred to either households or green firms. Each element of the matrix is the resulting emission in US or the euro area for a specific pair of tax parameters, τ_{EA} and τ_{US}. The emission level is defined as the unconditional mean of the emissions computed using a second-order approximation of the model.

D Tables

Table D.5: Differences in return on assets between brown and green companies

	Mean	Std Err.	2.5%	97.5%
EA green	4.89	.28	4.33	5.45
EA brown	4.94	.29	4.35	5.52
US green	6.56	.31	5.93	7.20
US brown	7.02	.23	6.55	7.50

Notes: statistics are computed on the sample of brown and green companies from GHG scope 1 and scope 2 database. Brown (green) companies are those at the top (bottom) 10% of the distribution of emissions. GHG scores ensure homogeneity in the business model and market segment of companies considered. To match firms in the model we consider only companies in the production of goods hence those in the "manufacturing", "consumers" and "materials" categories.

Table D.6: Calibrated parameters

Parameter	Value	Parameter	Value	Parameter	Value
β	0.9926	γ_r	0.75	ρ_ψ	0.5
β^*	0.9926	γ_r^*	0.75	ρ_ψ^*	0.5
ϕ	1	θ_π	1.2	$\rho_{A,g}$	0.5
ϕ^*	1	θ_π^*	1.2	$\rho_{A,g}^*$	0.5
σ_L	1	θ_Y	0.8	$\rho_{A,b}$	0.5
σ_L^*	1	θ_Y^*	0.8	$\rho_{A,b}^*$	0.5
ϕ^B	0.001	ξ	0.6	ρ_G	0.5
$\phi^{B,*}$	0.001	ξ^*	0.6	ρ_G^*	0.5
ϕ^K	1.728	ν	6	ρ_C	0.5
$\phi^{K,*}$	1.728	ν^*	6	ρ_C^*	0.5
δ	0.025	α	0.3	σ_G	0.01
δ^*	0.025	α^*	0.3	σ_G^*	0.01
ρ	0.33333	ψ	1	σ_r	0.01
ρ^*	0.33333	ψ^*	1	σ_r^*	0.01
ω	0.9	γ	0.304	$\sigma_{A,g}$	0.01
ω^*	0.9	γ	0.304	$\sigma_{A,g}^*$	0.01
Υ	0.5	ρ^x	0.9979	$\sigma_{A,b}$	0.01
Υ^*	0.5	$\rho^{x,*}$	0.9979	$\sigma_{A,b}^*$	0.01
$\frac{G}{Y}$	0.2	d_1	0.01	σ_C	0.01
$\frac{G}{Y}^*$	0.2	d_1^*	0.01	σ_C^*	0.01
τ_b	0	d_2	0.01	σ_ψ	0.01
τ_b^*	0	d_2^*	0.01	σ_ψ^*	0.01
τ_g	0	ρ_r	0.00		
τ_g^*	0	ρ_r^*	0.00		

Source: Eichenbaum et al. (2021).

Table D.7: Prior and posterior distributions

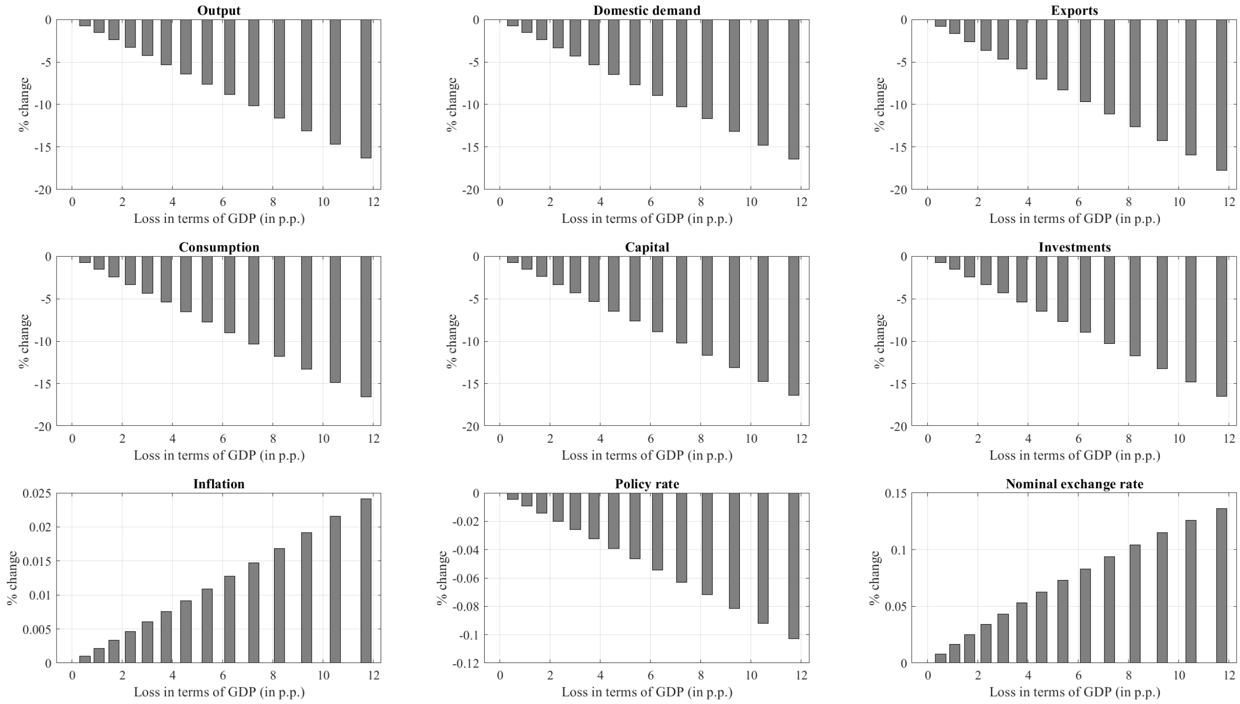
	Prior mean	Prior Std.	Prior	Post Mean	Post Std	[5% 95%]
			Parameters and shock processes			
ρ_A	0.5	0.1	Inv. Gamma	0.6629	0.01843	[0.6319 0.6939]
ρ_A^*	0.5	0.1	Inv. Gamma	0.5996	0.01451	[0.5752 0.624]
ρ_G	0.5	0.1	Inv. Gamma	0.6392	0.02369	[0.5994 0.679]
ρ_G^*	0.5	0.1	Inv. Gamma	0.4394	0.02388	[0.3992 0.4795]
ρ_C	0.5	0.1	Inv. Gamma	0.5598	0.0262	[0.5158 0.6038]
ρ_C^*	0.5	0.1	Inv. Gamma	0.4397	0.02007	[0.406 0.4734]
$\rho_{\mathcal{A}}$	0.5	0.1	Inv. Gamma	0.7073	0.0256	[0.6643 0.7503]
$\rho_{\mathcal{A}}^*$	0.5	0.1	Inv. Gamma	0.7048	0.01927	[0.6724 0.7371]
γ_r	0.75	0.1	Beta	0.7421	0.01369	[0.7191 0.7651]
γ_r^*	0.75	0.1	Beta	0.7543	0.02319	[0.7154 0.7933]
θ_π	1.2	0.2	Normal	1.294	0.03969	[1.228 1.361]
θ_π^*	1.2	0.2	Normal	1.538	0.03339	[1.481 1.594]
θ_y	0.6	0.1	Normal	0.5923	0.02576	[0.549 0.6356]
θ_y^*	0.6	0.1	Normal	0.6352	0.01752	[0.6057 0.6646]
ξ	0.6	0.1	Beta	0.8466	0.003307	[0.8411 0.8522]
ξ^*	0.6	0.1	Beta	0.8391	0.009179	[0.8237 0.8545]
			Climate model parameters			
γ	0.304	0.1	Normal	0.499	0.03525	[0.4398 0.5583]
γ^*	0.304	0.1	Normal	0.2698	0.02634	[0.2255 0.314]
d_1	0.01	0.01	Inv. Gamma	0.003967	0.001019	[0.002255 0.005678]
d_1^*	0.01	0.01	Inv. Gamma	0.01132	0.003067	[0.006166 0.01647]
d_2	0.01	0.01	Inv. Gamma	0.007615	0.001838	[0.004527 0.0107]
d_2^*	0.01	0.01	Inv. Gamma	0.00489	0.001078	[0.003079 0.006701]
			Standard deviations			
σ_r	0.010	2.00	Gamma	0.005004	0.000456	[0.004238 0.00577]
σ_r^*	0.010	2.00	Gamma	0.002454	0.0002569	[0.002022 0.002885]
σ_A	0.010	2.00	Gamma	0.01396	0.001391	[0.01162 0.01629]
σ_A^*	0.010	2.00	Gamma	0.02275	0.002653	[0.0183 0.02721]
σ_G	0.010	2.00	Gamma	0.1722	0.01842	[0.1413 0.2031]
σ_G^*	0.010	2.00	Gamma	0.104	0.01443	[0.07975 0.1282]
σ_C	0.010	2.00	Gamma	0.02698	0.00234	[0.02305 0.03091]
σ_C^*	0.010	2.00	Gamma	0.04417	0.003685	[0.03798 0.05036]
$\sigma_{\mathcal{A}}$	0.010	2.00	Gamma	0.009177	0.0009018	[0.007662 0.01069]
$\sigma_{\mathcal{A}}^*$	0.010	2.00	Gamma	0.007423	0.0006059	[0.006405 0.008441]

E Additional figures

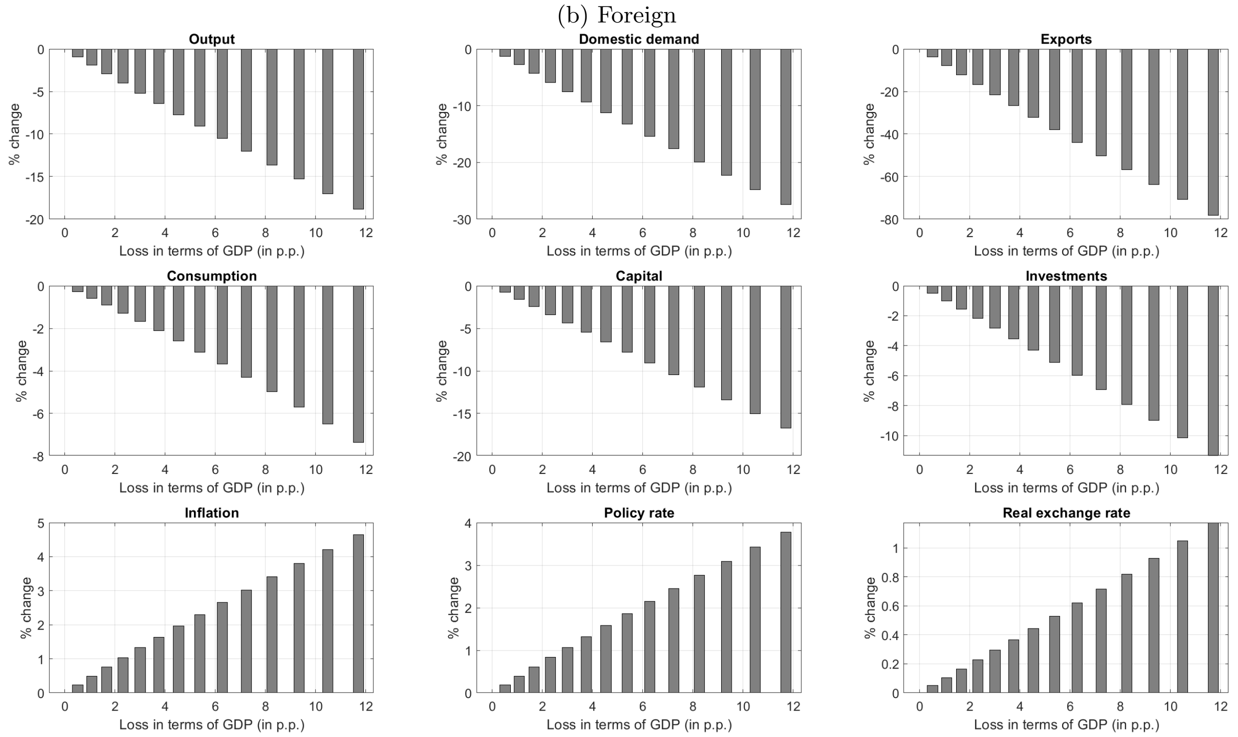
E.1 Calibrated baseline model

Figure E.8: Changes in the cumulative responses of endogenous variables to a 1% monetary policy shock

(a) Home

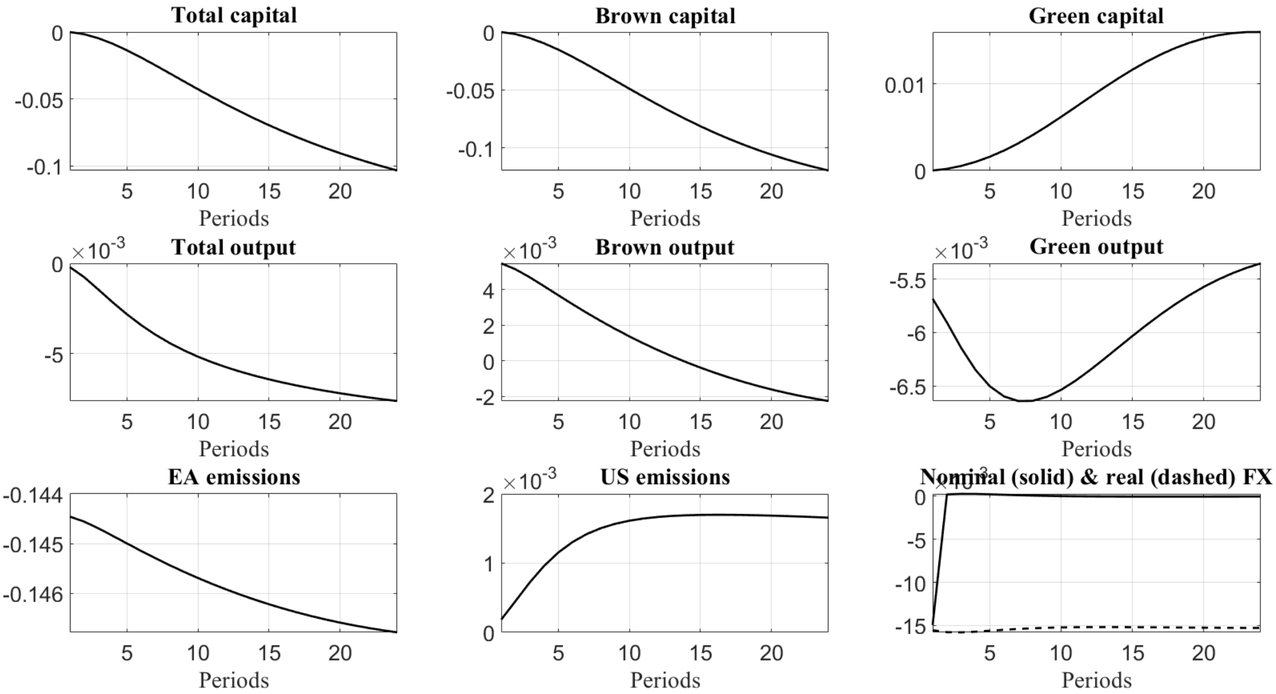


(b) Foreign



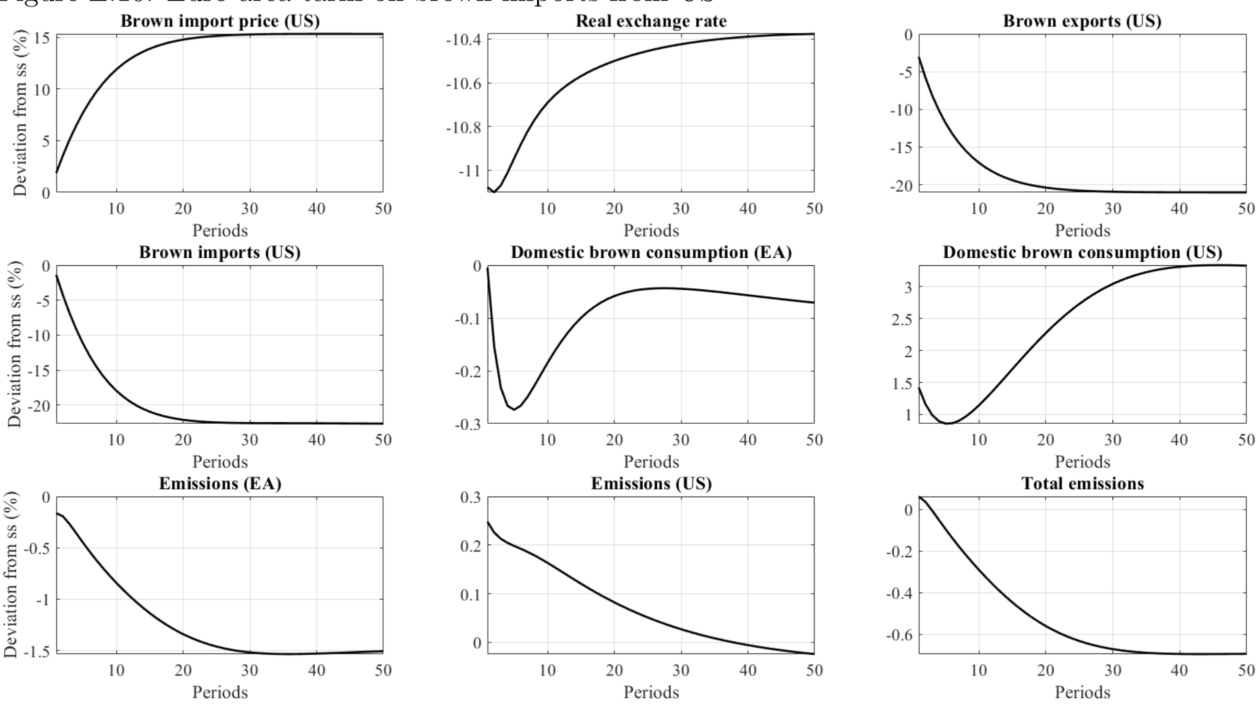
Notes: Changes are reported in percentage points relative to the model without the externality in production of the brown sector. Each bar corresponds to a different calibration of the loss function $\mathcal{L}(\cdot)$ that delivers a percent loss of steady state output reported on the x-axis.

Figure E.9: Impulse responses of a permanent 1% increase in carbon taxes in the euro area.



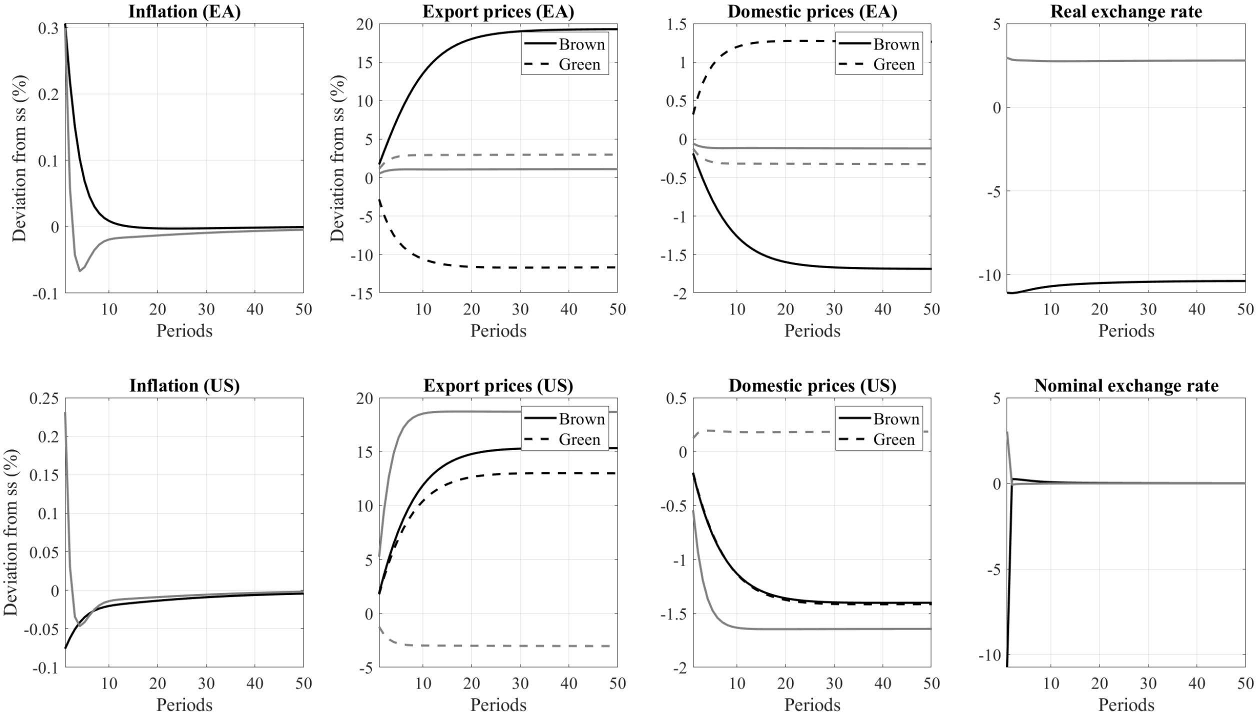
Notes: Impulse response, at first order, of a *permanent* 1% increase of brown taxation in the euro area. Responses are expressed in deviation from the steady state.

Figure E.10: Euro area tariff on brown imports from US



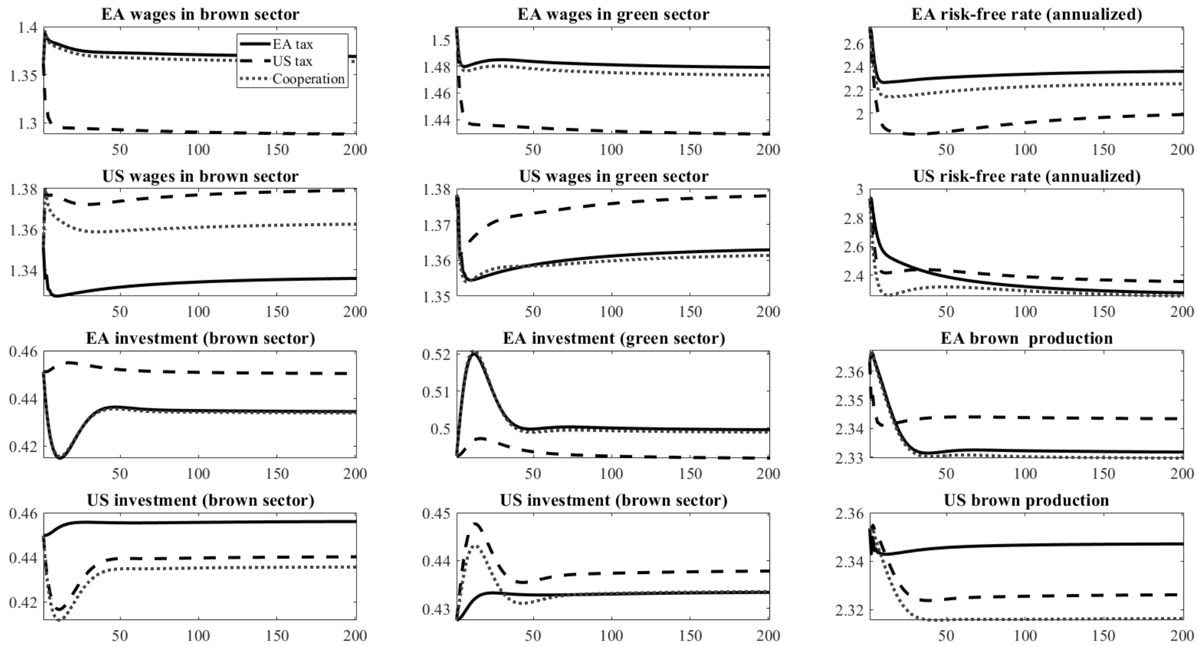
Notes: reaction to the permanent introduction of the optimal tariff in the US on euro area brown imports. Variables are expressed in percent deviations from the steady state.

Figure E.11: Comparison in the reaction to US tariffs under the calibrated and estimated model



Notes: reaction to the permanent introduction of the optimal tariff in the US on euro area brown imports. Variables are expressed in percent deviations from the steady state. Grey lines describe the reaction of variables in the symmetrically calibrated model, black lines refer to the estimated model.

Figure E.12: Transition of endogenous variables between *equilibria*



Notes: transition between the equilibrium without containment policy to the *equilibria* reported in Columns (1), (2) and (4) of Table 7. Variables are expressed in levels.